Formalizing Date Arithmetic and Statically Detecting Ambiguities for the Law

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ESOP
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Legal implementations

Some legal implementations are critical software: taxes, benefits

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Catala

- a DSL for computational laws

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- a DSL for computational laws
- providing transparency
- easing maintenance
- through interdisciplinary work

Computing dates

```
$ date -d "2024-01-31 + 1 month" +%F
```

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2024-03-02
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Non-monotonic behavior?!

A wide variety of date semantics

Different legal bodies and choices

- 1 month = 30 days (Council of European Communities)

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$\Longrightarrow$ Formal, flexible semantics required!

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- 1 month = 30 days (Council of European Communities)
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$\Longrightarrow$ Formal, flexible semantics required! Focus on Gregorian calendar.

Outline

Semantics

Semantics – Values

$$
\begin{array}{llll}
\text{values} & v & ::= & (y, m, d) \mid \bot \\
\text{date unit} & \delta & ::= & y \mid m \mid d \\
\text{expressions} & e & ::= & v \mid e \ +_\delta \ n
\end{array}
$$

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$$
\text{nb\_days}(y, m) = \begin{cases} 29 \text{ if } m = 2 \land \text{is\_leap}(y) \\ 28 \text{ if } m = 2 \land \neg\text{is\_leap}(y) \\ 30 \text{ if } m \in \{\, \texttt{Apr}, \texttt{Jun}, \texttt{Sep}, \texttt{Nov} \,\} \\ 31 \text{ otherwise} \end{cases}
$$

Semantics – invalid day number

Day additions with invalid day number propagate errors

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$$\textsc{Add-Days-Err1}\quad \frac{day < 1}{(y, m, day) +_d n \to \bot}$$

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$$\textsc{Add-Days-Err1}\quad \frac{day < 1}{(y, m, day) +_d n \to \bot}$$

$$\textsc{Add-Days-Err2}\quad \frac{day > \text{nb\_days}(y, m)}{(y, m, day) +_d n \to \bot}$$

Semantics – some cases of month addition

$$\textsc{Add-Month}\ \frac{1 \leq mo + n \leq 12}{(y, mo, d) +_m n \rightarrow (y, mo + n, d)}$$

Semantics – some cases of month addition

Add-Month

$$\frac{1 \leq mo + n \leq 12}{(y, mo, d) +_m n \rightarrow (y, mo + n, d)}$$

Add-Month-Over

$$\frac{mo + n > 12}{(y, mo, d) +_m n \rightarrow (y + 1, mo, d) +_m (n - 12)}$$

Semantics – some cases of month addition

$$\textsc{Add-Month}\quad \frac{1 \le mo + n \le 12}{(y, mo, d) +_m n \to (y, mo + n, d)}$$

$$\textsc{Add-Month-Over}\quad \frac{mo + n > 12}{(y, mo, d) +_m n \to (y + 1, mo, d) +_m (n - 12)}$$

Similar cases for ADD-MONTH-UNDER, year, day addition.

Semantics – Rounding

$$(2024, 01, 31) +_m 1 \to (2024, 02, 31)$$

Semantics – Rounding

$$(2024, 01, 31) +_m 1 \rightarrow (2024, 02, 31)$$

Rounding to valid dates required!

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$$\begin{array}{lcll} \text{rounding mode} & r & ::= & \uparrow \mid \downarrow \mid \bot \\ \text{expressions} & e & ::= & v \mid e +_\delta n \mid \text{rnd}_r\ e \end{array}$$

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$$\text{rnd}_\uparrow(2024, 02, 31) = (2024, 03, 01)$$

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Coreutils-like rounding not defined here

Semantics – Rounding

Round-Noop

$$\frac{1 \leq d \leq \text{nb\_days}(y, m)}{\text{rnd}_r(y, m, d) \rightarrow (y, m, d)}$$

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Round-Down

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Round-Up

$$\frac{d > \text{nb\_days}(y, m) \qquad (y, m, d) +_m 1 \xrightarrow{*} (y', m', d')}{\text{rnd}_\uparrow(y, m, d) \rightarrow (y', m', 1)}$$

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Round-Err2

$$\frac{d > \text{nb\_days}(y, m)}{\text{rnd}_\perp(y, m, d) \rightarrow \perp}$$

Semantics

Date-period addition

Given a period (ys, ms, ds):

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A date expression e is ambiguous iff $\mathrm{rnd}_\bot(e) \xrightarrow{*} \bot$

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iff roundings e yield different values

Formalized Properties

Non-properties

Commutativity of addition

$$(2024, 03, 31) +_{\uparrow} 1\text{m} +_{\uparrow} 1\text{d} = (2024, 05, 01) +_{\uparrow} 1\text{d} = (2024, 05, 02)$$

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All formalized with the F* proof assistant.

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Well-formedness

For any date d, any period p, any value v, and $r \in \{\downarrow, \uparrow\}$, we have:

$$\text{valid}(d) \wedge d +_r p \xrightarrow{*} v \Rightarrow \text{valid}(v)$$

Rounding-insensitivity Static Analysis

Meaningful ambiguities

Rounding choice can change comparisons

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- Considering product programs with both rounding modes

Delmas, Ouadjaout, and Miné. "Static Analysis of Endian Portability by Abstract Interpretation". SAS 2021

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- Considering product programs with both rounding modes
- Will reduce the need for costly legal interpretations

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$\mathbf{d}(d_1) \in [1, 31] \wedge \mathbf{m}(d_1) \in [1, 12] \wedge \mathbf{y}(d_1) = 2024$: all valid dates of 2024

Transfer function computing $(d, m, y) +^{\#}$ nb_m in abstract state abs

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let add_months ((d, m, y): var^3) (nb_m: int) (abs: state) =
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  (* Define exprs corresponding to the resulting month, year *)
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  (* Abstract switch with four different (guard, continuation) *)
  switch abs [
    (* Case 1: round resulting date in 30-day month *)
    d > 30 && is_one_of r_m [Apr;Jun;Sep;Nov], round 30 r_m r_y;






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    (* ~> continuation (assume guard) *)




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    d > 30 && is_one_of r_m [Apr;Jun;Sep;Nov], round 30 r_m r_y;
    (* Case 2: round resulting date to 28/02/Y, Y is not leap *)
    d > 28 && r_m = Feb && not (is_leap r_y), round 28 r_m r_y;
    (* Case 3: round resulting date to 29/02/Y, Y is leap *)
    d > 29 && r_m = Feb && is_leap r_y, round 29 r_m r_y;


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    d > 29 && r_m = Feb && is_leap r_y, round 29 r_m r_y;
    (* Case 4: no rounding *)
    mk_true, mk_date d r_m r_y;
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```

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4 cases apply, including:

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4 cases apply, including:

- 30-day month

$$\mathsf{d}(d1) = 31,\ \mathsf{m}(d1) \in \{\texttt{Mar}, \texttt{May}, \texttt{Aug}, \texttt{Oct}\}, \mathsf{d}(d2) = 30,\ \mathsf{m}(d2) = \mathsf{m}(d1) + 1, \mathsf{y}(d2) = \mathsf{y}(d1)$$

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- No rounding $\mathsf{d}(d1) = \mathsf{d}(d2),\ \mathsf{m}(\mathtt{d2}) \equiv_{12} \mathsf{m}(\mathtt{d1}) + 1, \mathsf{y}(d1) \leq \mathsf{y}(d2) \leq \mathsf{y}(d1) + 1$

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4 cases apply, including:

- 30-day month

$$\mathrm{d}(d1) = 31, \underbrace{\mathrm{m}(d1) \in \{\texttt{Mar}, \texttt{May}, \texttt{Aug}, \texttt{Oct}\}}_{\text{Bounded set of ints}}, \mathrm{d}(d2) = 30, \underbrace{\mathrm{m}(d2) = \mathrm{m}(d1) + 1, \mathrm{y}(d2) = \mathrm{y}(d1)}_{\text{Polyhedra}}$$

- No rounding $\mathrm{d}(d1) = \mathrm{d}(d2), \underbrace{\mathrm{m}(\mathtt{d2}) \equiv_{12} \mathrm{m}(\mathtt{d1}) + 1}_{\text{Linear congruence domain}}, \mathrm{y}(d1) \leq \mathrm{y}(d2) \leq \mathrm{y}(d1) + 1$

Abstract double semantics

Moving to double programs

- Analyze the program in both rounding modes

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- 30-day month

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$$\uparrow\!\mathsf{d}(d2) = 1, \uparrow\!\mathsf{m}(d2) \in \{\, \mathtt{May}, \mathtt{Jul}, \mathtt{Oct}, \mathtt{Dec} \,\}, \uparrow\!\mathsf{m}(d2) = \mathsf{m}(d1) + 2$$

$$\downarrow\!\mathsf{y}(d2) = \uparrow\!\mathsf{y}(d2) = \mathsf{y}(d1)$$

Implementation into Mopsa

- Open-source static analysis platform

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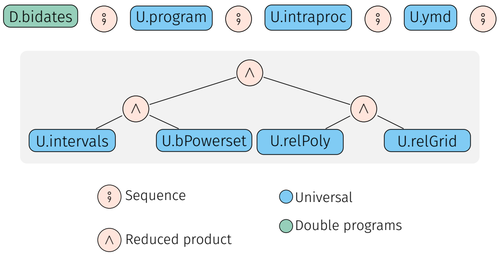

Extracted sample from French housing benefits

```
date current = rand_date();
date birthday = rand_date();
date intermediate = birthday + [2 years, 0 months, 0 days];
date limit = first_day_of(intermediate);
assert(sync(current < limit));
```

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1 date current = rand_date();
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```

```
5: assert(sync(current < limit));
               ^^^^^^^^^^^^^^^

Desynchronization detected: (current < limit). Hints:
↑month(limit) = 3, ↑day(limit) = 1, ↓month(limit) = 2, ↓day(limit) = 1,
↑month(intermediate) = 3, ↑day(intermediate) = 1, ↓month(intermediate) = 2,
↓day(intermediate) = 28, month(birthday) = 2, day(birthday) = 29,
year(birthday) =[4] 0, month(current) = 2, day(current) = [1,29],
year(current) = ↑year(intermediate) = ↑year(limit)
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Desynchronization detecte
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Computed, actual counter-example

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Case Study: French Housing Benefits

Catala, a DSL for computational laws

Article D823-20 of the French building regulations

The moving allowance is awarded to individuals or households with at least three children born or to be born and who move into a new home entitled to one of the personal housing allowances during a period between the first day of the calendar month following the third month of pregnancy for a child of rank three or more and the last day of the month preceding that in which the child reaches his or her second birthday.
This allowance is payable if the right to assistance is acquired within six months of the date of moving in.

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scope MovingAllowanceEligibility:
 definition condition_moving_period under condition
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  -- MoreThan3Children of date_of_birth_or_pregnancy:
   (match date_of_birth_or_pregnancy with pattern
    -- DateOfBirth of birthday
     current_date < (first_day_of_month of (birthday + 2 year))
    # ...
   )
  )
consequence fulfilled
```
````


Merigoux, Chataing, and Protzenko. “Catala: a programming language for the law”. 2021
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- Literate programming
- Lawyer-developer duos
- Default logic tailored to the law
- Housing benefits: 20kLoC (incl. law)

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- Date-rounding library `dates-calc`

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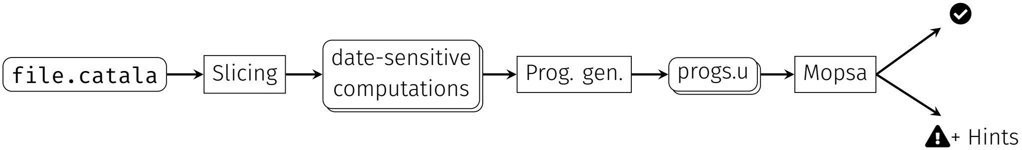
Date ambiguity detection pipeline



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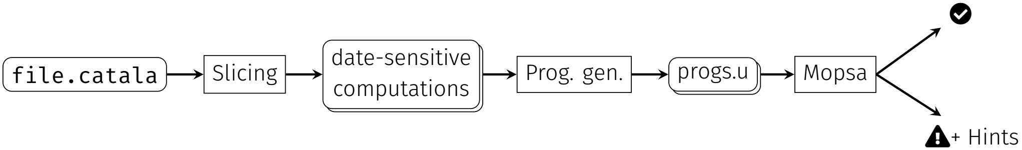

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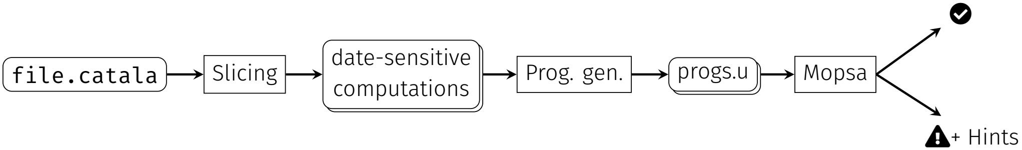

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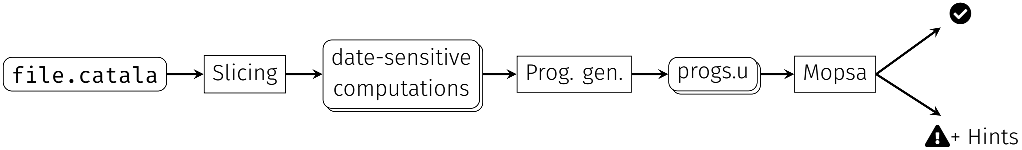

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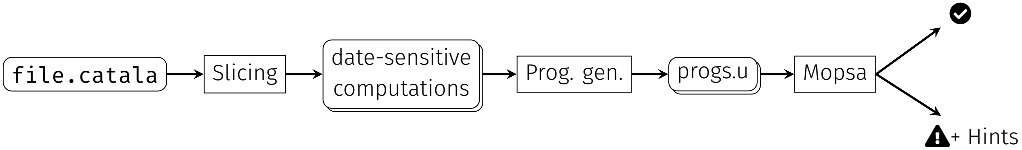

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Manual inter-scope extraction

16 additional cases:

- 10 can be proved safe assuming `current_date` $\geq$ `2023`
- Other are real issues

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Related Work

Survey of implementations

- Java, `boost` round down
- Python `stdlib`: no month addition
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Floating-point arithmetic

- FP widely used & more complex!
- Different rounding modes
- No analysis of rounding-sensitivity?

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“**A**utomatic **V**erification **o**f **Cat**ala programs” (AVoCat) project funded by Inria