Static Analysis by Abstract Interpretation Collecting Types of Python Programs

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Under the supervision of Antoine Miné

Introduction

Software Bugs

Year 2000
Ariane 5
software bugs
Patriot
HeartBleed
Boeing 787
OpenSSL
Nasa Mariner 1
Mars Global Surveyor
Amazon
Bank of Queensland
Soviet Gas
Deutsche Telekom
Kerberos
PayPal
Pentium
Zenit 3SL
Delta-3
AT&T
A400M
Chinook
Hamburg trains
Nasa Spirit
Mars Climate Orbiter
Internet Explorer
Northeast Blackout
Hartford Coliseum
Skyshield 35
Titan IV
Therac-25
F-16
F-22
Osprey
Medtronic
Toyota
Hanover trains
Mars Pathfinder
Mars Polar Lander
USS Yorktown
Phobos 1
Year 2000
Hitomi
CryoSat-1
Nasa Mariner 1
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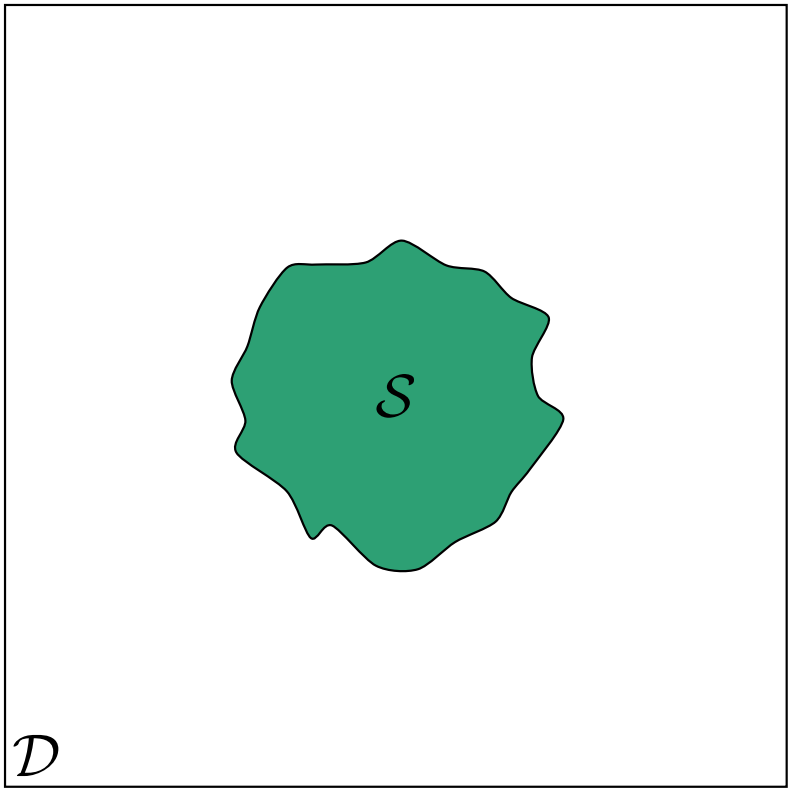
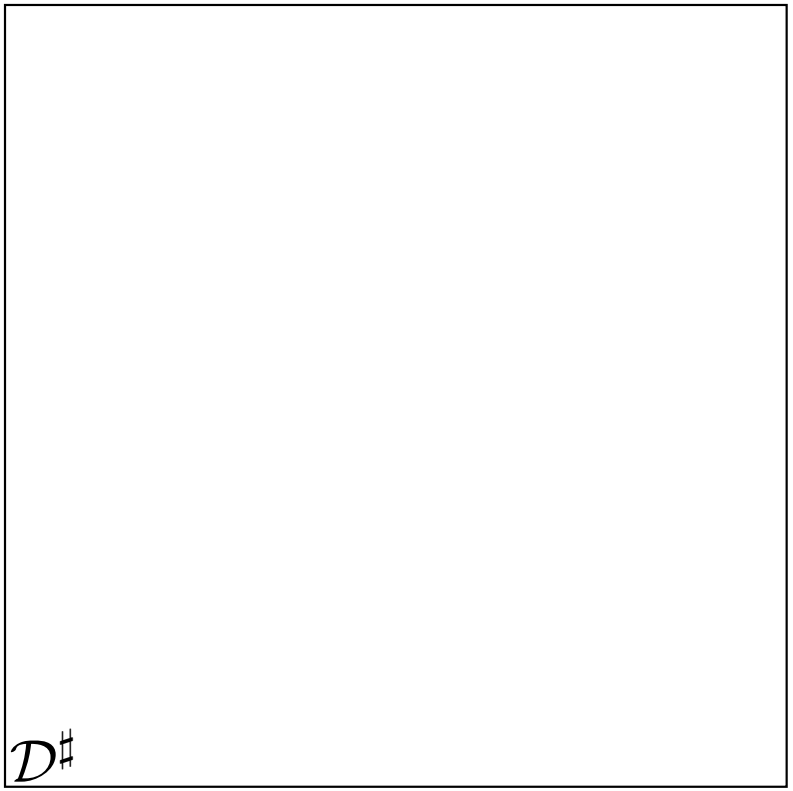
Abstract Interpretation





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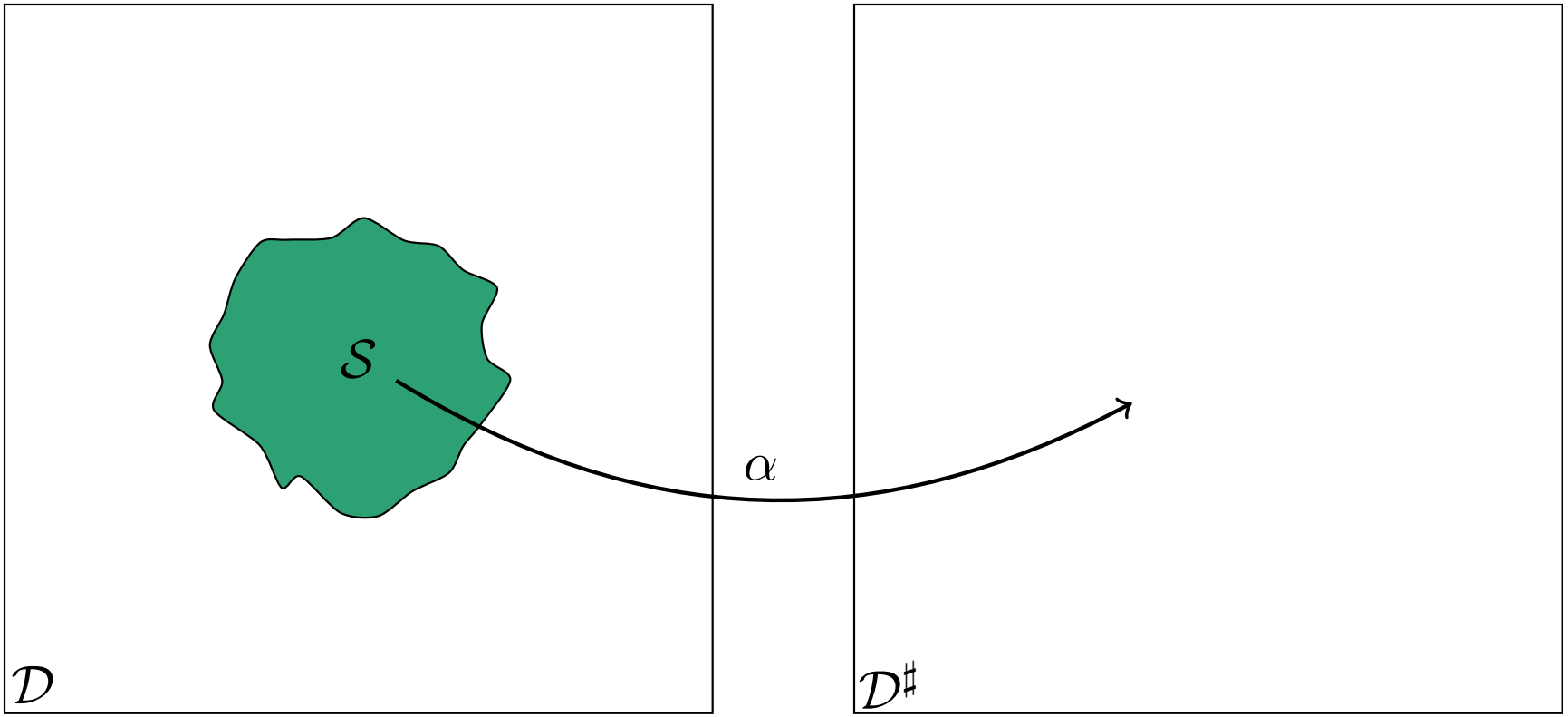

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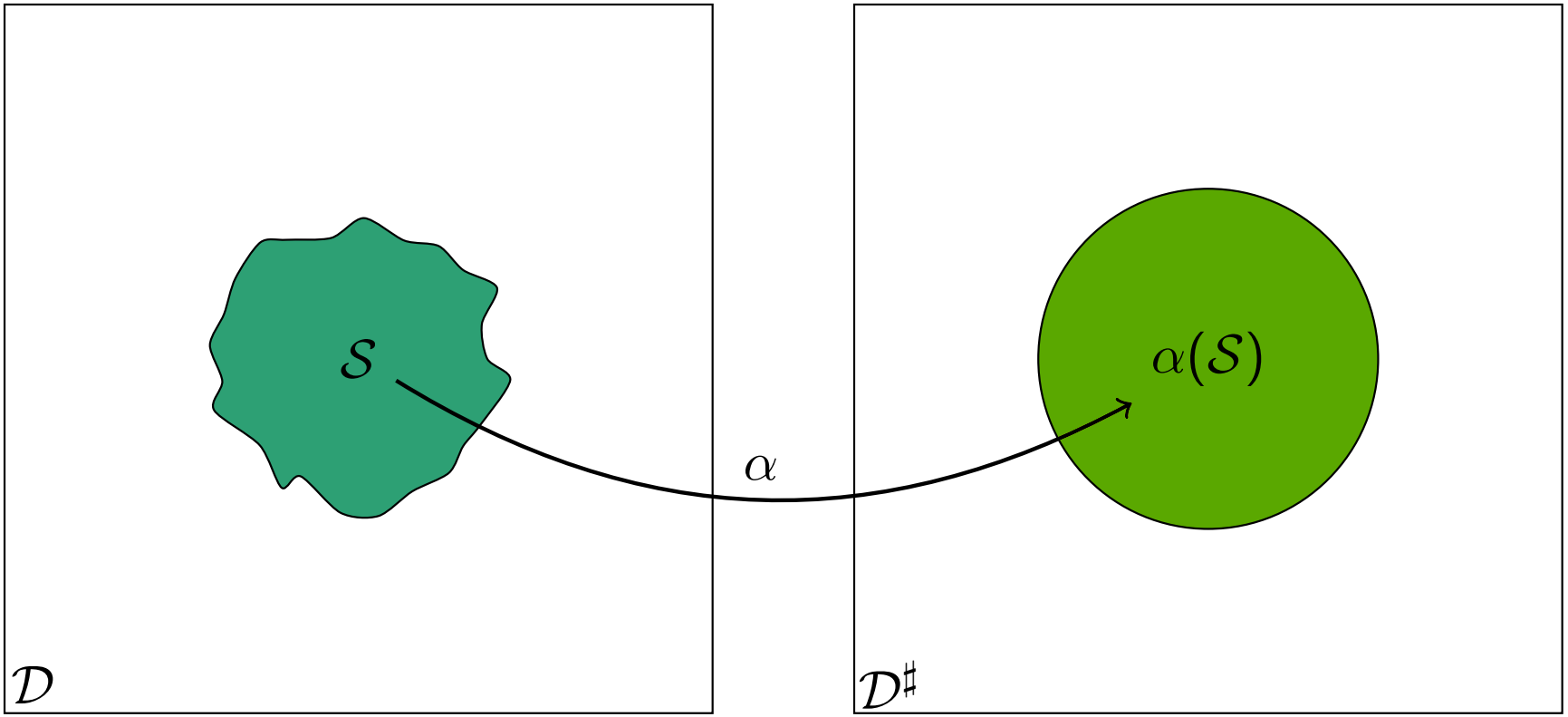

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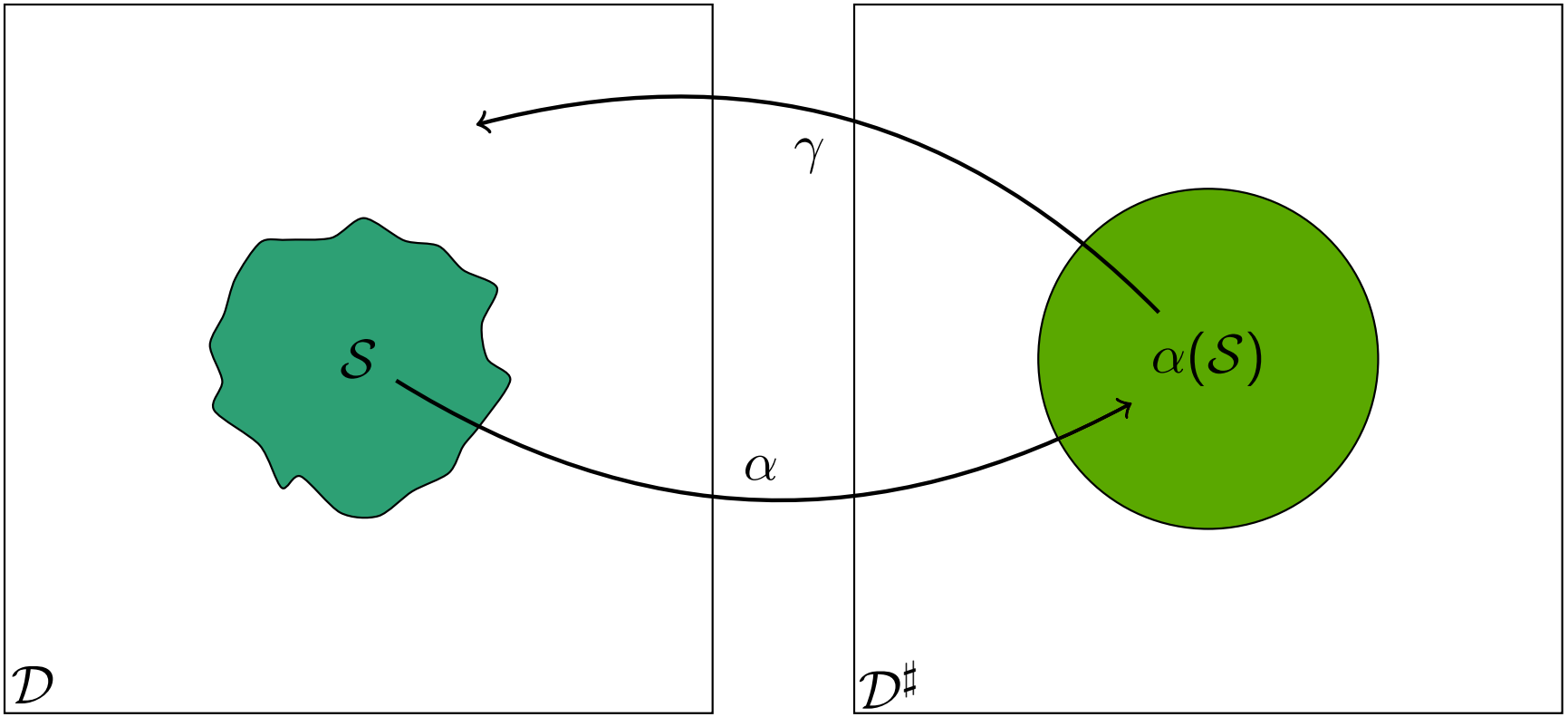

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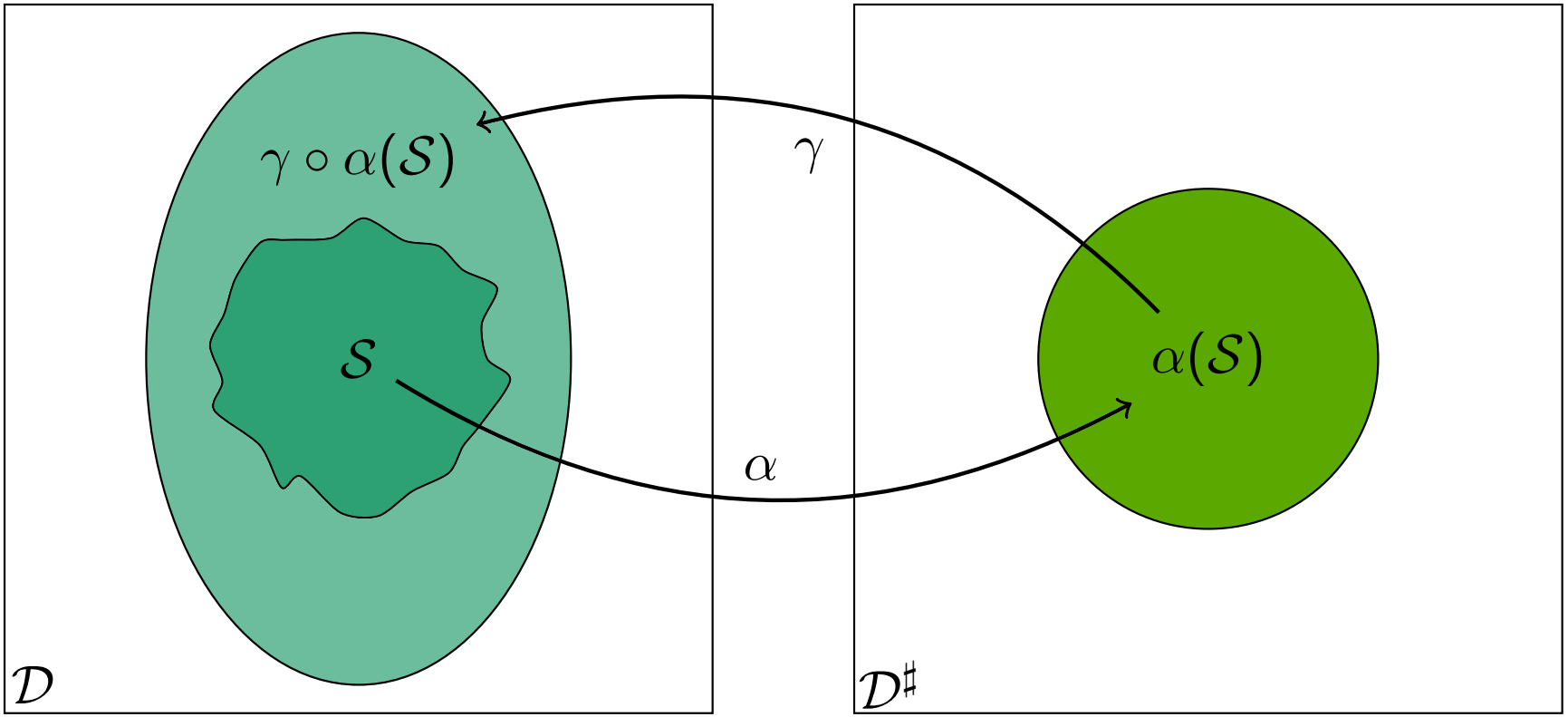
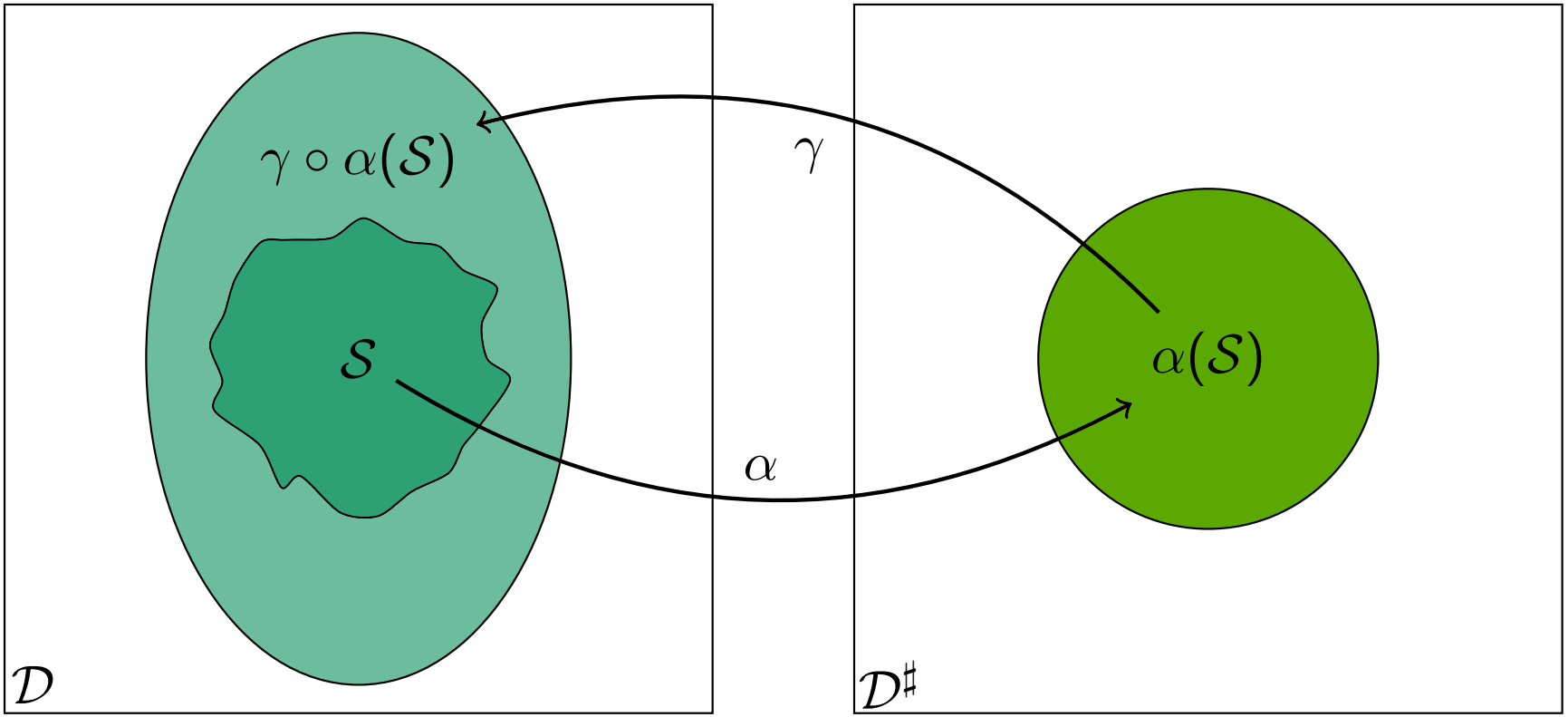


Abstract Interpretation – Numerical Invariants



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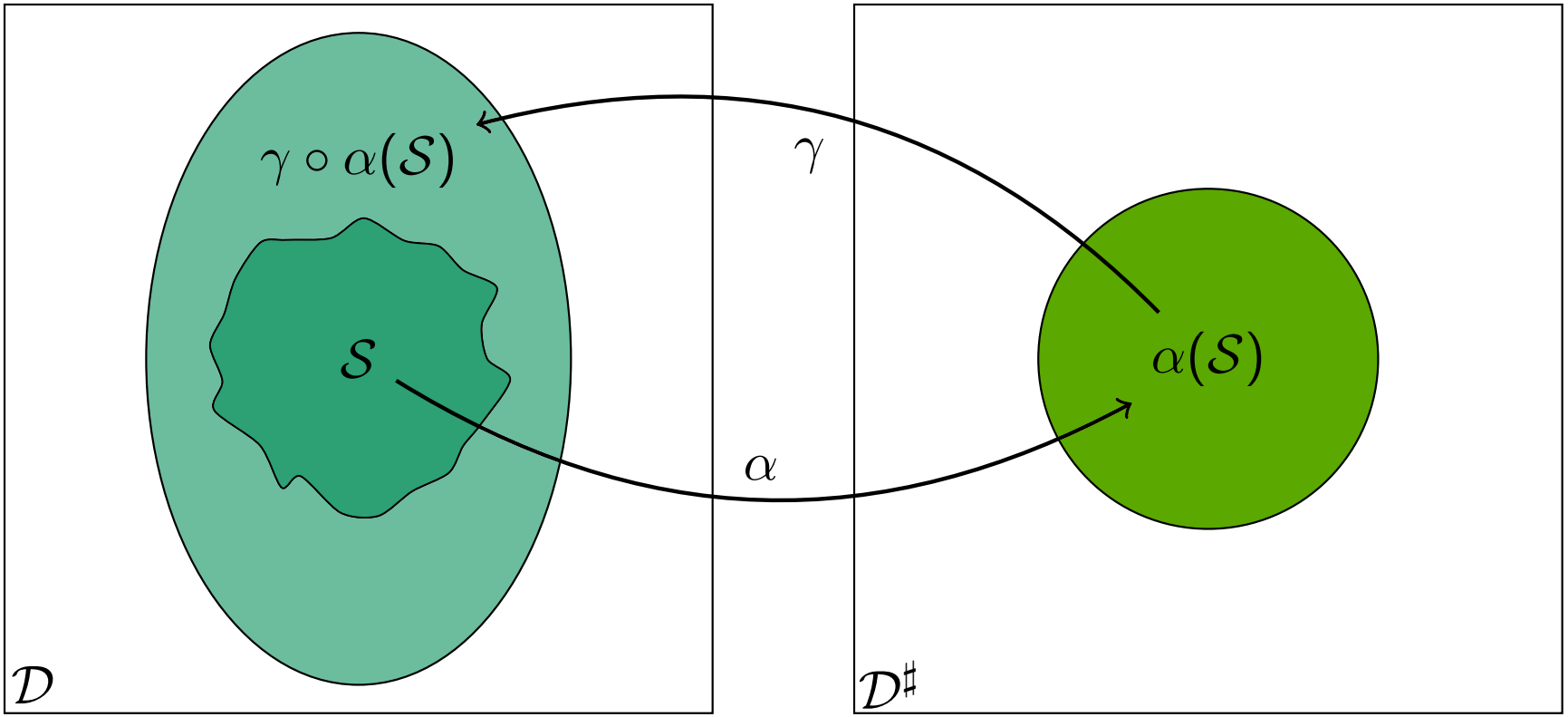

- $\mathcal{D} = \mathcal{P}(\mathbb{Z})$
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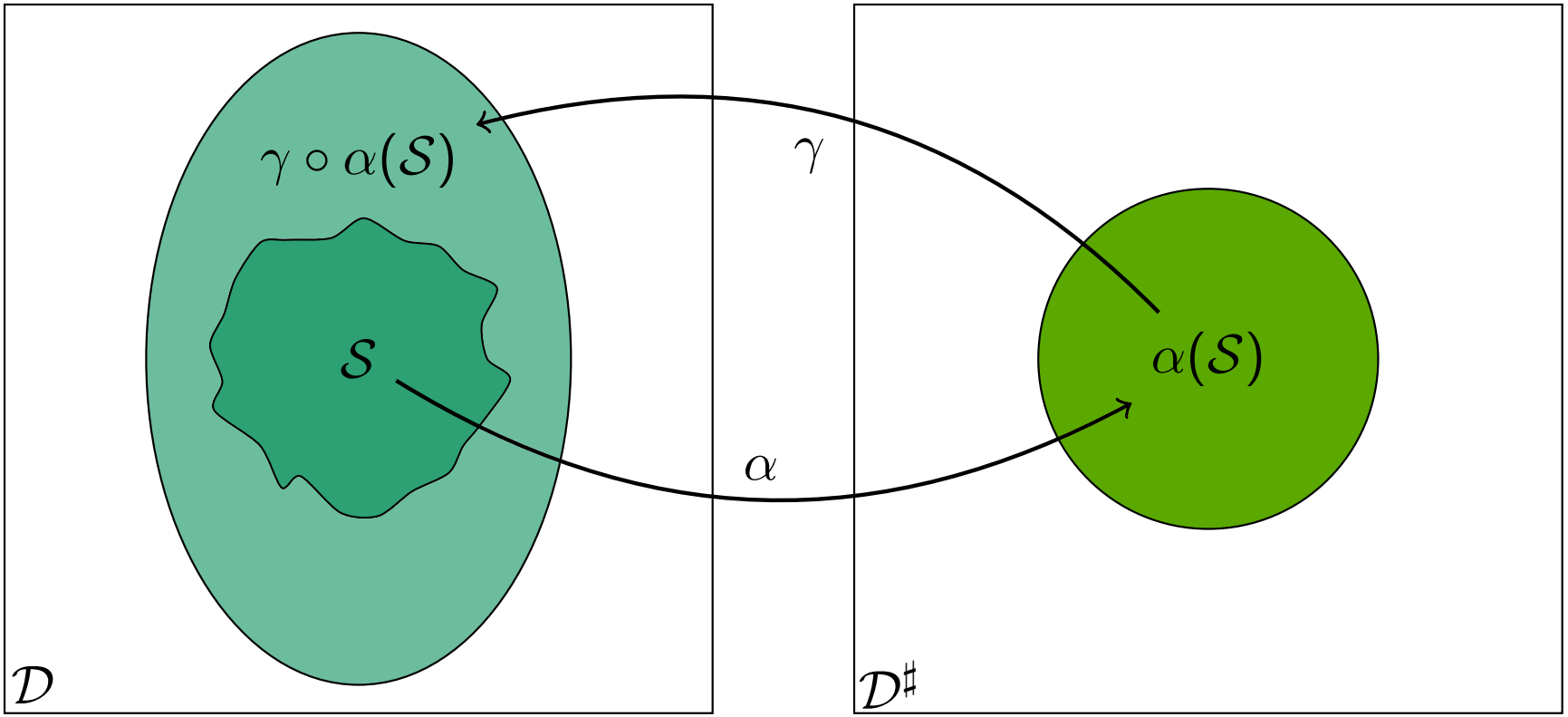

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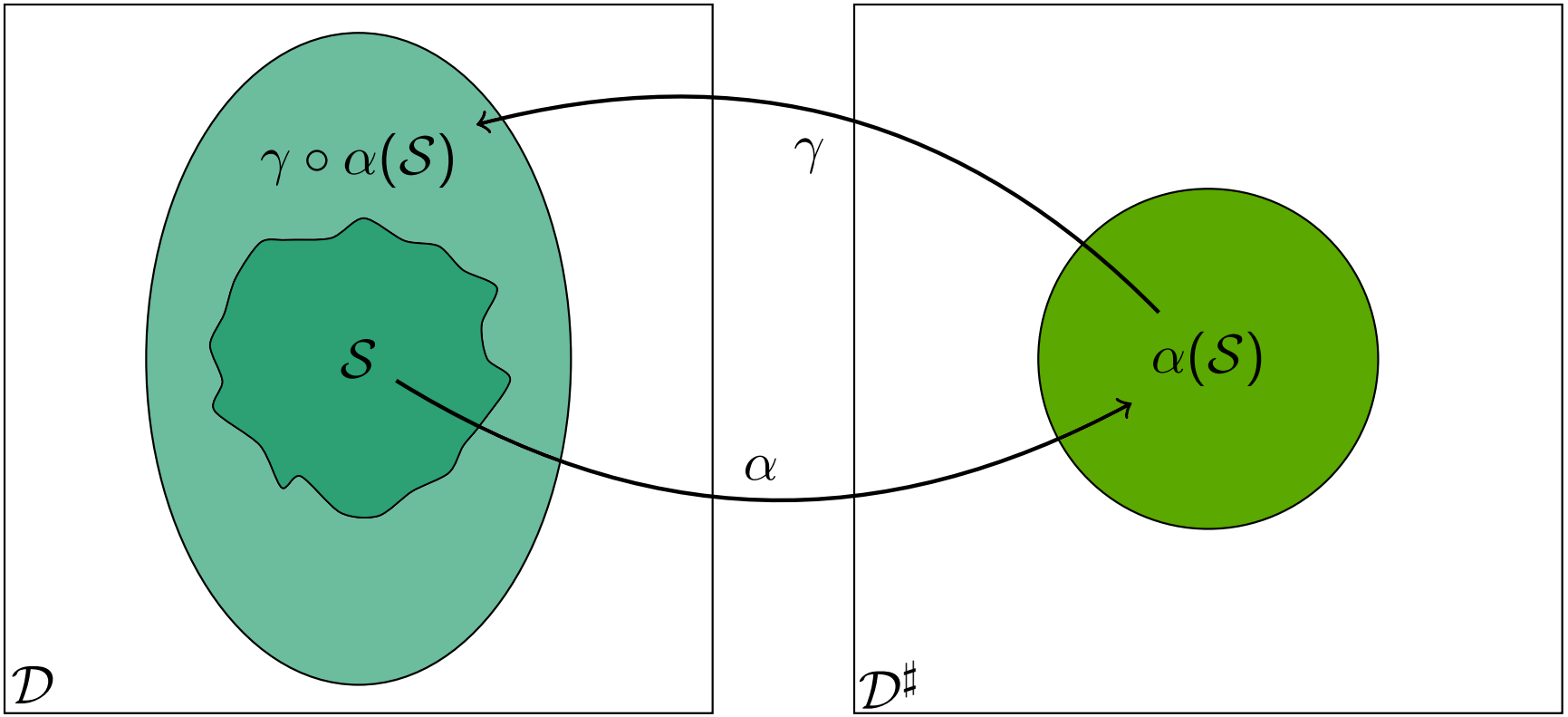

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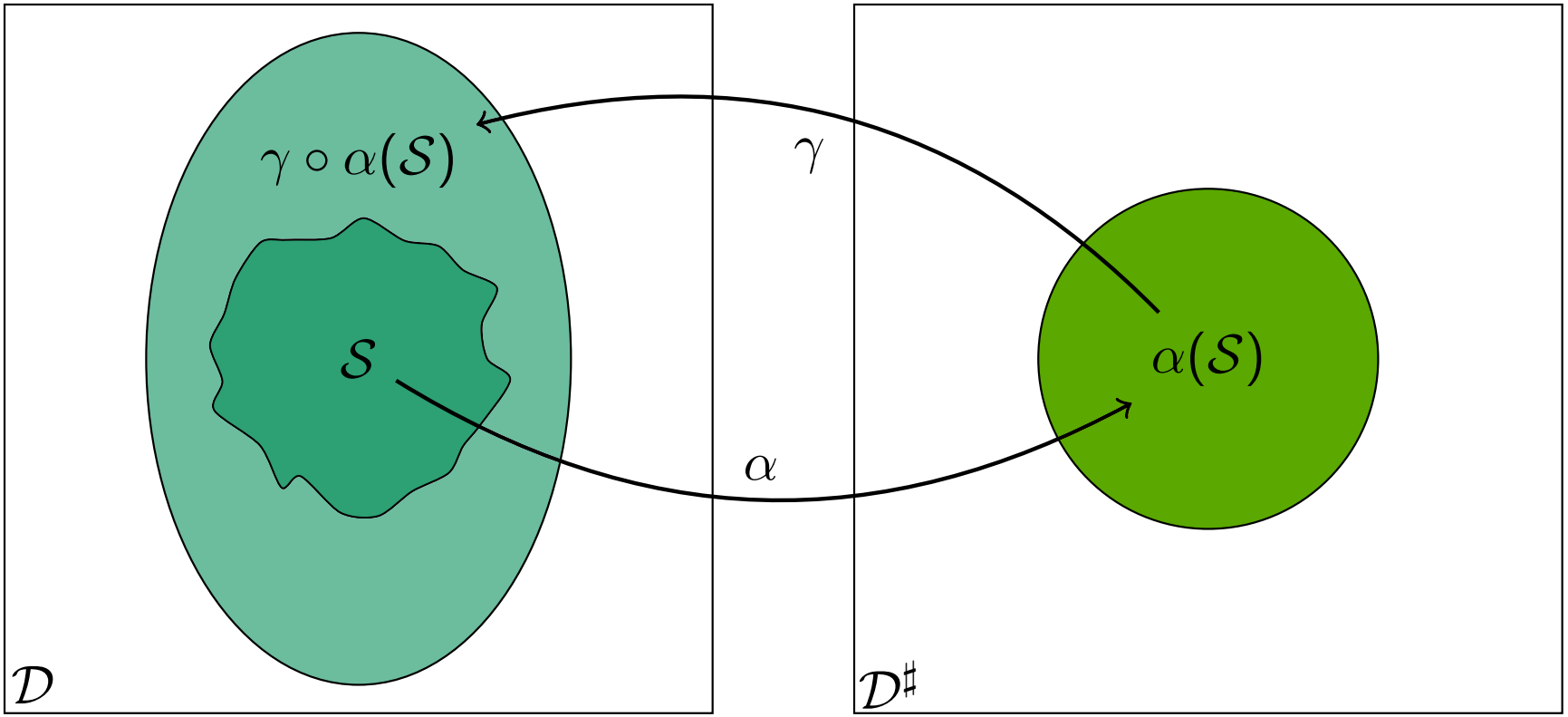

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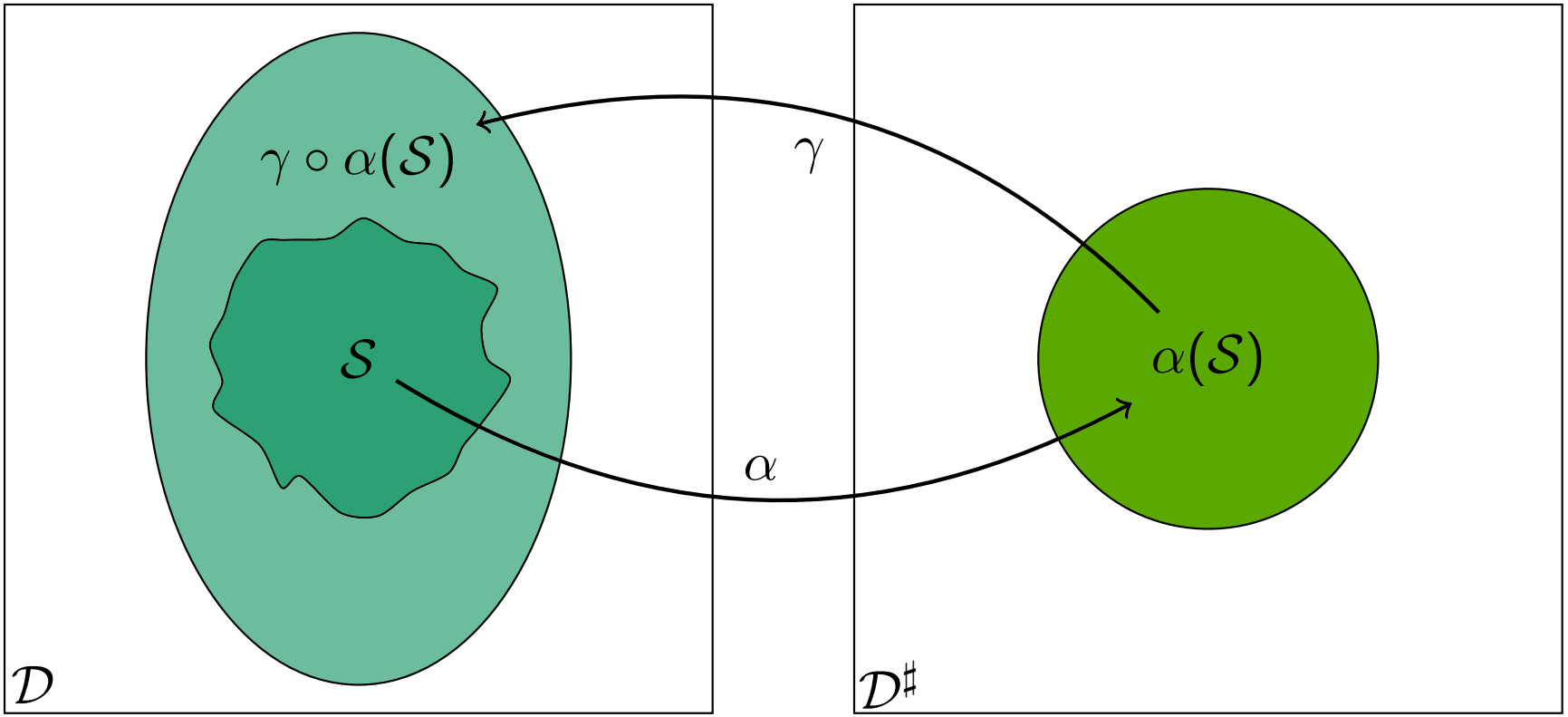
- $\gamma \circ \alpha(\mathcal{S}) = \{\, 1, 2, 3, 4, 5 \,\}$

Abstract Interpretation – Typing of the λ-calculus



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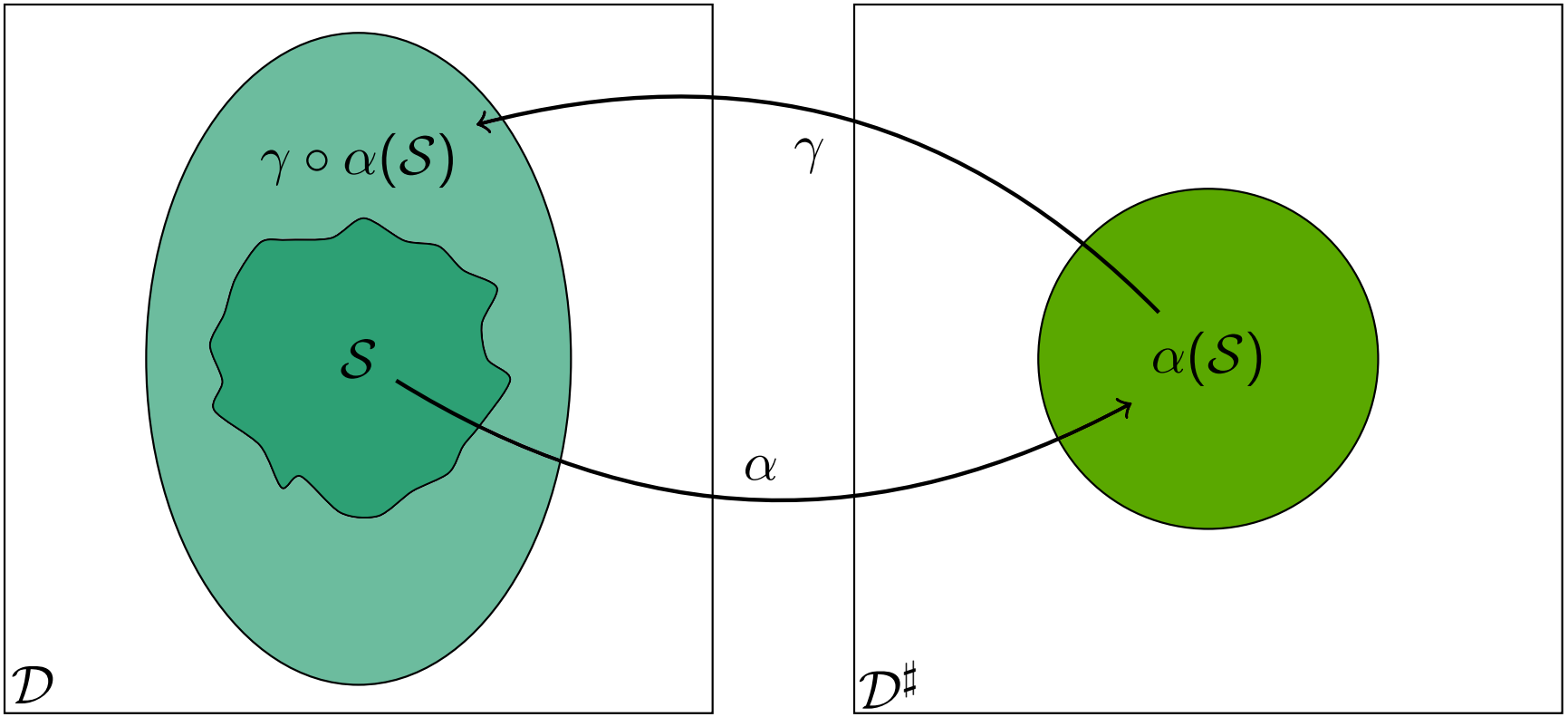

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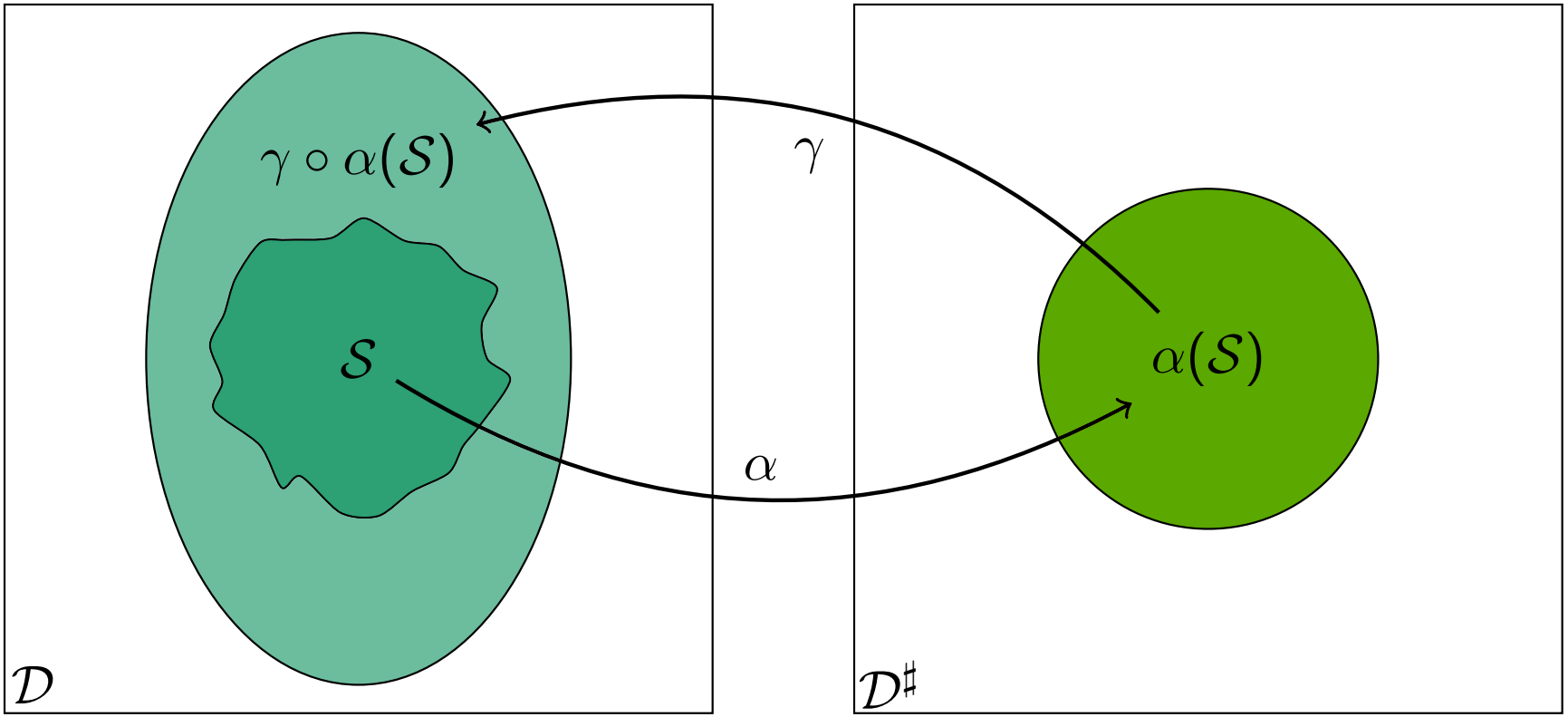

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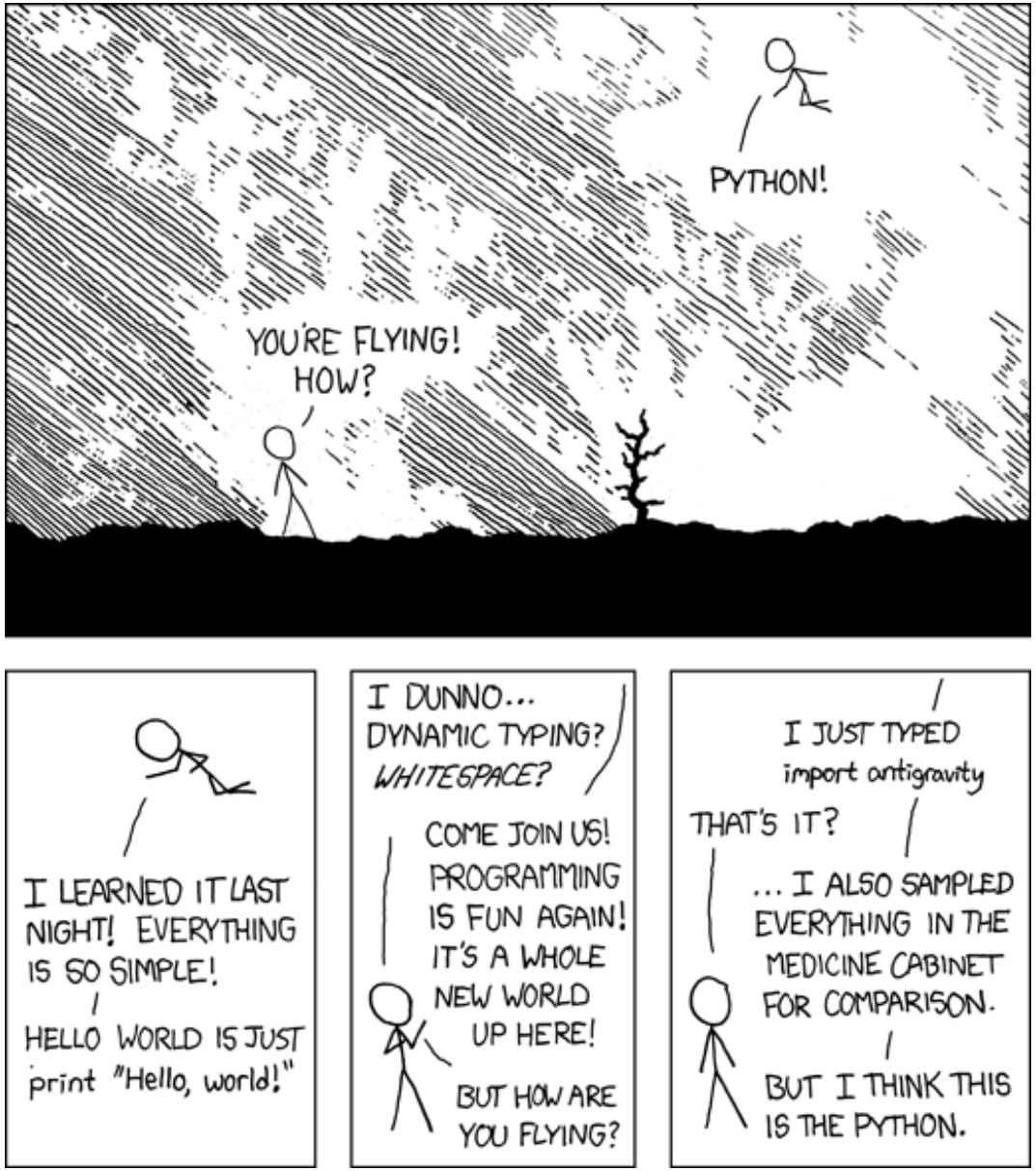
$\Longrightarrow$ more in the report

Python



Source: xkcd (https://xkcd.com/353/)

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- used in the teaching and scientific communities
- used for popular projects: Django, SageMath

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Two Notions of Typing

- nominal typing (`isinstance`)
- duck typing (`hasattr`)

Python

fspath returns "the file system representation of the path."

Excerpt (and simplification) from Python's stdlib (os.py:1022).

```
def fspath(p):
  if isinstance(p, (str, bytes)):
    return p
  elif hasattr(p, "__fspath__"):
    res = p.__fspath__()
    if isinstance(res, (str, bytes)):
      return res
    else:
      raise TypeError("...")
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Objects having method $__\mathit{fspath}__$, itself returning type α

Ideas and Inspirations

Static analyses are especially helpful on dynamic languages.

Change Python's value analysis[1] into:

- A more abstract analysis (types as abstraction of values)
- A more precise analysis (relationality)

[1] Fromherz, Ouadjaout, and Miné. "Static Value Analysis of Python Programs by Abstract Interpretation". NASA Formal Methods - 10th International Symposium, NFM 2018, Newport News, VA, USA, April 17-19, 2018, Proceedings.

Goals

- Collecting types and type errors for Python programs. Contrary to a type system:
  - Naturally flow-sensitive analysis.
  - No rejection of untypable programs.
- Performing a sound analysis: no alarm $\implies$ no bug.
- Performing a relational analysis, supporting bounded parametric polymorphism.
- Accepting all Python semantics (no restriction such as "variables have a fixed type").

Concrete Semantics of Python

Concrete Semantics of Python

- ✗ No formal semantics (and no standard) defined for Python.
- CPython's implementation defines the semantics.
- We use a slight modification of the semantics of Fromherz, Ouadjaout, and Miné[2].

[2] Fromherz, Ouadjaout, and Miné. "Static Value Analysis of Python Programs by Abstract Interpretation". NASA Formal Methods - 10th International Symposium, NFM 2018, Newport News, VA, USA, April 17-19, 2018, Proceedings.

Semantics of +

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\begin{array}{l}
\mathbb{E}[\![\, e_1 + e_2 \,]\!]\,(f, \epsilon, \sigma) \stackrel{\text{def}}{=} \\
\quad \text{if } f \neq cur \text{ then } (f, \epsilon, \sigma, addr_{\texttt{None}}) \text{ else} \\
\quad \bullet\ \text{letif } (f_1, \epsilon_1, \sigma_1, a_1) = \mathbb{E}[\![\, e_1 \,]\!]\,(f, \epsilon, \sigma) \text{ in } \bullet \\
\quad \text{letif } (f_2, \epsilon_2, \sigma_2, a_2) = \mathbb{E}[\![\, e_2 \,]\!]\,(f_1, \epsilon_1, \sigma_1) \text{ in} \\
\quad \text{if } has_field(a_1, __add__, \sigma_2) \text{ then} \\
\qquad \text{letif } (f_3, \epsilon_3, \sigma_3, a_3) = \mathbb{E}[\![\, a_1.__add__(a_2) \,]\!]\,(f_2, \epsilon_2, \sigma_2) \text{ in} \\
\qquad \text{if } \sigma_3(a_3) = (_, \texttt{NotImpl}) \text{ then} \\
\qquad\quad \text{if } has_field(a_2, __radd__, \sigma_3) \wedge typeof(a_1) \neq typeof(a_2) \text{ then} \\
\qquad\qquad \text{letif } (f_4, \epsilon_4, \sigma_4, a_4) = \mathbb{E}[\![\, a_2.__radd__(a_1) \,]\!]\,(f_3, \epsilon_3, \sigma_3) \text{ in} \\
\qquad\qquad \text{if } \sigma_4(a_4) = (_, \texttt{NotImpl}) \text{ then } \texttt{TypeError}(f_4, \epsilon_4, \sigma_4) \\
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\qquad \text{else } f_3, \epsilon_3, \sigma_3, a_3 \\
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\end{array}
$$

Evaluate e_1

slight modification from Fromherz, Ouadjaout, and Miné, "Static Value Analysis of Python Programs by Abstract Interpretation"

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\end{array}
$$

Evaluate e_2

slight modification from Fromherz, Ouadjaout, and Miné, "Static Value Analysis of Python Programs by Abstract Interpretation"

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$\quad$ if $f \neq cur$ then $(f, \epsilon, \sigma, addr_{\texttt{None}})$ else

$\quad$ letif $(f_1, \epsilon_1, \sigma_1, a_1) = \mathbb{E}[\![\, e_1 \,]\!]\,(f, \epsilon, \sigma)$ in

$\quad$ letif $(f_2, \epsilon_2, \sigma_2, a_2) = \mathbb{E}[\![\, e_2 \,]\!]\,(f_1, \epsilon_1, \sigma_1)$ in

$\quad$ ● if $has_field(a_1, __add__, \sigma_2)$ then ●

$\quad\quad$ letif $(f_3, \epsilon_3, \sigma_3, a_3) = \mathbb{E}[\![\, a_1.__add__(a_2) \,]\!]\,(f_2, \epsilon_2, \sigma_2)$ in

$\quad\quad$ if $\sigma_3(a_3) = (_, \texttt{NotImpl})$ then

$\quad\quad\quad$ if $has_field(a_2, __radd__, \sigma_3) \wedge typeof(a_1) \neq typeof(a_2)$ then

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$\quad\quad\quad\quad$ if $\sigma_4(a_4) = (_, \texttt{NotImpl})$ then $\texttt{TypeError}(f_4, \epsilon_4, \sigma_4)$

$\quad\quad\quad\quad$ else $(f_4, \epsilon_4, \sigma_4, a_4)$

$\quad\quad\quad$ else $\texttt{TypeError}(f_3, \epsilon_3, \sigma_3)$

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If e_1 has method $__add__$

slight modification from Fromherz, Ouadjaout, and Miné, "Static Value Analysis of Python Programs by Abstract Interpretation"

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$\quad$ if $has_field(a_1, __add__, \sigma_2)$ then

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and if $__add__$ does not return `NotImpl`

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\quad \text{if } has_field(a_1, __add__, \sigma_2) \text{ then} \\
\qquad \text{letif } (f_3, \epsilon_3, \sigma_3, a_3) = \mathbb{E}[\![\, a_1.__add__(a_2) \,]\!]\,(f_2, \epsilon_2, \sigma_2) \text{ in} \\
\qquad \text{if } \sigma_3(a_3) = (_, \texttt{NotImpl}) \text{ then} \\
\qquad\quad \text{if } has_field(a_2, __radd__, \sigma_3) \wedge typeof(a_1) \neq typeof(a_2) \text{ then} \\
\qquad\qquad \text{letif } (f_4, \epsilon_4, \sigma_4, a_4) = \mathbb{E}[\![\, a_2.__radd__(a_1) \,]\!]\,(f_3, \epsilon_3, \sigma_3) \text{ in} \\
\qquad\qquad \text{if } \sigma_4(a_4) = (_, \texttt{NotImpl}) \text{ then } \texttt{TypeError}(f_4, \epsilon_4, \sigma_4) \\
\qquad\qquad \text{else } (f_4, \epsilon_4, \sigma_4, a_4) \\
\qquad\quad \text{else } \texttt{TypeError}(f_3, \epsilon_3, \sigma_3) \\
\qquad \bullet \text{else } f_3, \epsilon_3, \sigma_3, a_3 \bullet \\
\quad \text{else if } has_field(a_2, __radd__, \sigma_2) \wedge typeof\, a_1 \neq typeof\, a_2 \text{ then} \\
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\qquad \text{else } (f_3, \epsilon_3, \sigma_3, a_3) \\
\quad \text{else } \texttt{TypeError}(f_2, \epsilon_2, \sigma_2)
\end{array}
$$

return the result of the $__add__$ call

slight modification from Fromherz, Ouadjaout, and Miné, "Static Value Analysis of Python Programs by Abstract Interpretation"

Semantics of +

$\mathbb{E}[\![\, e_1 + e_2 \,]\!]\,(f, \epsilon, \sigma) \stackrel{\text{def}}{=}$

$\quad$ if $f \neq cur$ then $(f, \epsilon, \sigma, addr_{\texttt{None}})$ else

$\quad$ letif $(f_1, \epsilon_1, \sigma_1, a_1) = \mathbb{E}[\![\, e_1 \,]\!]\,(f, \epsilon, \sigma)$ in

$\quad$ letif $(f_2, \epsilon_2, \sigma_2, a_2) = \mathbb{E}[\![\, e_2 \,]\!]\,(f_1, \epsilon_1, \sigma_1)$ in

$\quad$ if $has_field(a_1, __add__, \sigma_2)$ then

$\quad\quad$ letif $(f_3, \epsilon_3, \sigma_3, a_3) = \mathbb{E}[\![\, a_1.__add__(a_2) \,]\!]\,(f_2, \epsilon_2, \sigma_2)$ in

$\quad\quad$ if $\sigma_3(a_3) = (_, \texttt{NotImpl})$ then

$\quad\quad\quad$ ● if $has_field(a_2, __radd__, \sigma_3) \wedge typeof(a_1) \neq typeof(a_2)$ then ●

$\quad\quad\quad\quad$ letif $(f_4, \epsilon_4, \sigma_4, a_4) = \mathbb{E}[\![\, a_2.__radd__(a_1) \,]\!]\,(f_3, \epsilon_3, \sigma_3)$ in

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$\quad\quad\quad\quad$ else $(f_4, \epsilon_4, \sigma_4, a_4)$

$\quad\quad\quad$ else $\texttt{TypeError}(f_3, \epsilon_3, \sigma_3)$

$\quad\quad$ else $f_3, \epsilon_3, \sigma_3, a_3$

● else if $has_field(a_2, __radd__, \sigma_2) \wedge typeof\, a_1 \neq typeof\, a_2$ then ●

$\quad\quad$ letif $(f_3, \epsilon_3, \sigma_3, a_3) = \mathbb{E}[\![\, a_2.__radd__(a_1) \,]\!]\,(f_2, \epsilon_2, \sigma_2)$ in

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$\quad\quad$ else $(f_3, \epsilon_3, \sigma_3, a_3)$

$\quad$ else $\texttt{TypeError}(f_2, \epsilon_2, \sigma_2)$

otherwise, if e_1 and e_2 have different types

slight modification from Fromherz, Ouadjaout, and Miné, "Static Value Analysis of Python Programs by Abstract Interpretation"

Semantics of +

$$
\begin{array}{l}
\mathbb{E}[\![\, e_1 + e_2 \,]\!]\,(f, \epsilon, \sigma) \stackrel{\text{def}}{=} \\
\quad \text{if } f \neq cur \text{ then } (f, \epsilon, \sigma, addr_{\texttt{None}}) \text{ else} \\
\quad \text{letif } (f_1, \epsilon_1, \sigma_1, a_1) = \mathbb{E}[\![\, e_1 \,]\!]\,(f, \epsilon, \sigma) \text{ in} \\
\quad \text{letif } (f_2, \epsilon_2, \sigma_2, a_2) = \mathbb{E}[\![\, e_2 \,]\!]\,(f_1, \epsilon_1, \sigma_1) \text{ in} \\
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\qquad \text{else } (f_3, \epsilon_3, \sigma_3, a_3) \\
\quad \text{else } \texttt{TypeError}(f_2, \epsilon_2, \sigma_2)
\end{array}
$$

call the reflected method $__radd__$

slight modification from Fromherz, Ouadjaout, and Miné, "Static Value Analysis of Python Programs by Abstract Interpretation"

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$\quad\quad$ else $(f_3, \epsilon_3, \sigma_3, a_3)$

$\quad$ else $\texttt{TypeError}(f_2, \epsilon_2, \sigma_2)$

raise a type error if nothing works

slight modification from Fromherz, Ouadjaout, and Miné, "Static Value Analysis of Python Programs by Abstract Interpretation"

Type Abstract Domain

Types: what do we need?

Types: what do we need?

Top and bottom types

Types: what do we need?

Top and bottom types

Class declarations

Types: what do we need?

Top and bottom types

Class declarations

Instances

```
class A: pass
x = A()

if *:
  x.o = 3
  x.u = 4
else:
  x.u = 4
ze = x.o
z = x.u
```

Types: what do we need?

Top and bottom types

Class declarations

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```

$$\Longrightarrow \mathit{Instance}[v, \mathit{mustAttributes}, \mathit{mayAttributes}]$$

Types: what do we need?

Type variables

- `l = True if * else 42`

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$$\implies \alpha \in V_\Sigma$$

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- `l = [1, 2]` $\rightsquigarrow \mathit{List}[\mathit{int}]$

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$$\implies \mathit{List}[v], v \in V^\sharp$$

Types

Polymorphic types

$$
\begin{aligned}
V^\sharp = \bot \mid \top \\
&\mid c, c \in \mathit{Classes} \\
&\mid \mathit{List}[v] \mid \mathit{Tuple}[v] \mid \ \ldots, v \in V^\sharp \\
&\mid \mathit{Instance}[v, \mathit{mustAttributes}, \mathit{mayAttributes}], v \in V^\sharp \\
&\mid V_\Sigma \in \{\alpha, \beta, \ldots\}
\end{aligned}
$$

Monormophic types

$$V_m^\sharp = V^\sharp \text{ without type variables}$$

Structure of the Abstract Domain

$\mathcal{D}_1^\sharp$ partitions the environments in packs of variables having the same type. Partitions are refined to take aliasing into account.

$$\mathcal{D}^\sharp = \mathcal{D}_1^\sharp \times \mathcal{D}_2^\sharp \times \mathcal{D}_3^\sharp$$

$$\mathcal{D}_1^\sharp = \mathrm{Id} \rightharpoonup \mathit{TypeId} \times \mathit{Aliasing}$$

$$\mathcal{D}_2^\sharp = \mathit{TypeId} \rightharpoonup V^\sharp$$

$$\mathcal{D}_3^\sharp = V_\Sigma \rightharpoonup \mathcal{P}(V_m^\sharp)$$

Example

$$d_1 = x \mapsto (0, \bot), y \mapsto (0, \bot), z \mapsto (1, \bot)$$

Structure of the Abstract Domain

$\mathcal{D}_2^\sharp$ associates to each partition number a polymorphic type.

$$\mathcal{D}^\sharp = \mathcal{D}_1^\sharp \times \mathcal{D}_2^\sharp \times \mathcal{D}_3^\sharp$$
$$\mathcal{D}_1^\sharp = \mathbf{Id} \rightharpoonup \mathit{Typeld} \times \mathit{Aliasing}$$
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Example

$$\begin{cases} d_1 & = x \mapsto (0, \bot), y \mapsto (0, \bot), z \mapsto (1, \bot) \\ d_2 & = 0 \mapsto \alpha, 1 \mapsto \mathit{List}[\alpha] \end{cases}$$

Structure of the Abstract Domain

$\mathcal{D}_3^\sharp$ is a type variable environment.

$$\mathcal{D}^\sharp = \mathcal{D}_1^\sharp \times \mathcal{D}_2^\sharp \times \mathcal{D}_3^\sharp$$
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Example

$$\begin{cases} d_1 & = x \mapsto (0, \bot), y \mapsto (0, \bot), z \mapsto (1, \bot) \\ d_2 & = 0 \mapsto \alpha, 1 \mapsto \mathit{List}[\alpha] \\ d_3 & = \alpha \mapsto \{\, \mathit{int}, \mathit{float} \,\} \end{cases}$$

Concretizations & Abstract Operators

Concretization:

$$\gamma : \mathcal{D}^\sharp \to \mathcal{P}(\mathcal{E} \times \mathcal{H})$$

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Abstract Operators:

- Subset testing
- Join ($\sqcup$)
- Widening
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  - instances
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Abstract Operators:

- Subset testing
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- Widening
- Filtering
  - instances
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Soundness:

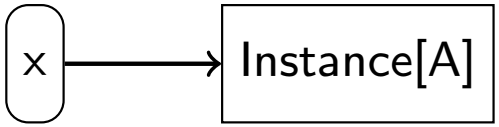
$$\forall d, d' \in \mathcal{D}^\sharp, \gamma(d) \cup \gamma(d') \subseteq \gamma(d \sqcup d')$$

Examples

Examples

```
class A: pass

x = A()
if *:
  y = x
else:
  y = A()
y.a = 2
z = x.a
```



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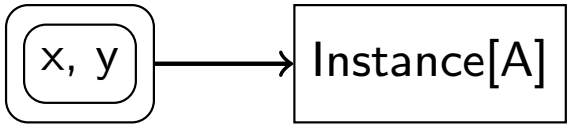

$$\begin{cases} d_1 & = x \mapsto (0, a), y \mapsto (0, a) \\ d_2 & = 0 \mapsto \mathit{Instance}[A] \\ d_3 & = _ \end{cases}$$

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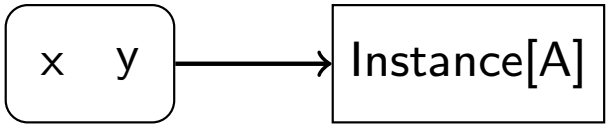

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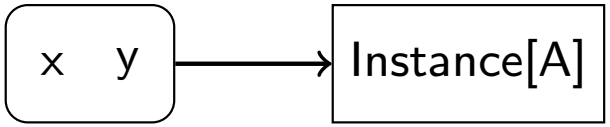

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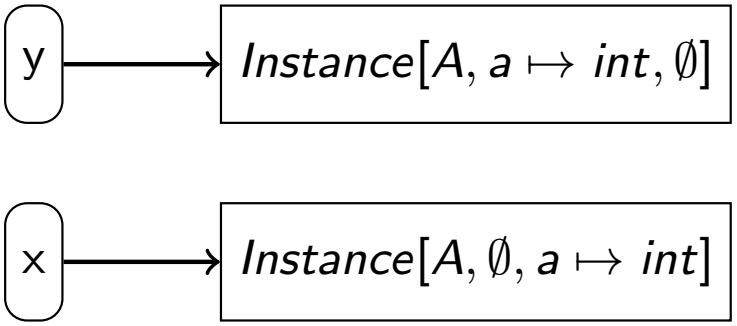

$$\begin{cases} d_1 & = y \mapsto (0, \bot), x \mapsto (1, \bot) \\ d_2 & = 0 \mapsto \mathit{Instance}[A, a \mapsto \mathit{int}, \emptyset], \\ & \quad 1 \mapsto \mathit{Instance}[A, \emptyset, a \mapsto \mathit{int}] \\ d_3 & = _ \end{cases}$$

Examples

```
class A: pass

x = A()
if *:
  y = x
else:
  y = A()
y.a = 2
z = x.a
```

Attribute Error:

$$x \mapsto \mathit{Instance}[A, \emptyset, a \mapsto \mathit{int}]$$

$$\begin{cases} d_1 & = y \mapsto (0, \bot), x \mapsto (1, \bot), \mathit{exn_var} \mapsto (2, \bot) \\ d_2 & = \ldots, 2 \mapsto \mathit{Instance}[\mathit{AttributeError}] \\ d_3 & = _ \end{cases}$$

Examples

```
def fspath(p):
  if isinstance(p, (str, bytes)):
    return p
  elif hasattr(p, "__fspath__"):
    res = p.__fspath__()
    if isinstance(res, (str, bytes)):
      return res
    else:
      raise TypeError("...")
  else:
    raise TypeError("...")

r1 = fspath('a')
r2 = fspath(FSPath())
r3 = fspath(b'path')
```

Interprocedural analysis?

Examples

```
def fspath(p):
  if isinstance(p, (str, bytes)):
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Interprocedural analysis?

- Inlining

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Interprocedural analysis?

- Inlining
- Bottom-up analysis (as in OCaml)

Examples

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- Inlining
- Bottom-up analysis (as in OCaml)
- Summary-based, top-down analysis

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$$\Longrightarrow \; Summary[f, (InputArgs, OutputArgs)\ list] \in V^{\sharp}$$

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$Summary[fspath, []]$

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$Summary[fspath, [(str, str)]]$

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$$F = Instance[FSPpath, [__fspath__], \emptyset]$$

$$Summary[fspath, [\quad (str, str) \quad (F, str)]]$$

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$$F = \mathit{Instance}[\mathit{FSPpath}, [__\mathit{fspath}__], \emptyset]$$

$$\begin{aligned} \mathit{Summary}[\mathit{fspath}, [\quad & (\mathit{str}, \mathit{str}) \\ & (F, \mathit{str}) \\ & (\mathit{bytes}, \mathit{bytes})]] \end{aligned}$$

$\rightsquigarrow$

$$\begin{aligned} \mathit{Summary}[\mathit{fspath}, [\quad & (\alpha, \alpha), \alpha \in \{\, \mathit{str}, \mathit{bytes} \,\} \\ & (F, \mathit{str})]] \end{aligned}$$

Implementation

Implementation into MOPSA

Modular **O**pen **P**latform for **S**tatic **A**nalysis.

- Modular abstract domains for: abstract values, control-flow, ...
- Statements flow through abstract domains until one answers
- User selects the combination of abstract domains
- Currently, subsets of C and Python are supported

Implementation into MOPSA

In the type analysis, the following abstract domains are used:

- Generic control-flow ones
- One handling function calls through inlining
- Generic ones handling Python objects, class, functions, ...

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About 1500 lines of added OCaml.

✓ I was able to reuse some abstract domains.

✗ Some abstract domains were not modular enough, I generalized them or wrote alternatives.

Related Work

Other Type Analyses

Program	Fritz and Hage	Pytype	Typpete	MOPSA - types
Analysis method	Dataflow analysis	Unclear	SMT-solver	AI
class_attr_ok	✓	✗	✳	✓
class_pre_store	✓	✓	✓	✓
default_args_class	✓	✓	✓	✓
except_clause	✗	✳	✓	✓
fspath	✗	✗	✳	✓
magic	✓	✓	✓	✳
polyfib	✳	✗	✳	✳
poly_lists	✳	✓	✳	✓
vehicle	✓	✓	✓	✳
widening	✓	✗	✳	✓

✓ sound and precise

✳ sound but false alarm

✗ unsound

Conclusion

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- Design of a static type analysis by abstract interpretation
- Relational abstract domain, supporting bounded parametric polymorphism
- Concretization function, giving concrete meaning to the abstract domain

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Future work:

- Implementation (summary-based analysis, larger Python support)
- Combining this analysis with the value analysis

Appendix

Non-Goals

We are not designing a type system.

- No rejection of programs, only error collection.

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We are not designing a type system.

- No rejection of programs, only error collection.
- Abstract interpretation based soundness proof.
- Our analysis is naturally flow-sensitive.
- Our analysis is not as modular as most type systems are.

Environments

$$\mathcal{E} \stackrel{\text{def}}{=} \textbf{Id} \rightharpoonup \textbf{Addr} \cup \texttt{Undef}$$

$$\mathcal{H} \stackrel{\text{def}}{=} \textbf{Addr} \rightharpoonup \textbf{Obj} \times \textbf{Val}$$

$$\mathcal{F} \stackrel{\text{def}}{=} \{\, \mathit{cur}, \mathit{ret}, \mathit{brk}, \mathit{cont}, \mathit{exn} \,\}$$

$$\textbf{Id} \subseteq \mathit{string}$$

$$\textbf{Obj} \stackrel{\text{def}}{=} \mathit{string} \rightharpoonup \textbf{Addr}$$

$$\textbf{Val} \stackrel{\text{def}}{=} \mathbb{Z} \cup \mathit{string} \cup \{\texttt{True}, \texttt{False}, \texttt{None}, \texttt{NotImpl}\}$$

---

slight modification from Fromherz, Ouadjaout, and Miné, "Static Value Analysis of Python Programs by Abstract Interpretation"

Examples

```
class A: pass

x = A()
if *:
  y = x
else:
  y = A()
y.a = 2
try:
  z = x.a
except AttributeError:
  z = y.a
```

$$exn, \begin{cases} d_1 & = y \mapsto (0, \bot), x \mapsto (1, \bot), exn_var \mapsto (2, \bot) \\ d_2 & = \ldots, 2 \mapsto Instance[AttributeError] \\ d_3 & = _ \end{cases}$$

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$$cur, \begin{cases} d_1 & = y \mapsto (0, \bot), x \mapsto (1, \bot) \\ d_2 & = 0 \mapsto Instance[A, a \mapsto int, \emptyset], \\ & \quad 1 \mapsto Instance[A, \emptyset, a \mapsto int] \\ d_3 & = _ \end{cases}$$

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Value Analysis of Python

Value Analysis[3]:

- incomparable with our analysis (due to the relationality)

[3] Fromherz, Ouadjaout, and Miné. "Static Value Analysis of Python Programs by Abstract Interpretation". NASA Formal Methods - 10th International Symposium, NFM 2018, Newport News, VA, USA, April 17-19, 2018, Proceedings.

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Value Analysis of Python

Value Analysis[3]:

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- generally more precise information, more costly
- support for large Python library more difficult to implement

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Other Type Analyses

- Typpete [4]: uses an SMT solver to perform type inference

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- Pytype: from engineers at Google, no technical reference

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