

A Multilanguage Static Analysis of Python/C Programs with Mopsa

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Introduction

Static Program Analysis

average.py

```
1 def average(l):
2     m = 0
3     for i in range(len(l)):
4         m = m + l[i]
5         m = m // (i + 1)
6     return s
7
8 r1 = average([1, 2, 3])
9 r2 = average(['a', 'b', 'c'])
```

TypeError: unsupported operand type(s) for '+': 'int' and 'str'

argslen.c

```
1 #include <string.h>
2
3 int main(int argc, char *argv[]) {
4     int i = 0;
5     for (char **p = argv; *p; p++) {
6         strlen(*p); // valid string
7         i++; // no overflow
8     }
9     return 0;
10 }
```

No alarm

Specifications of the analyzer

Inference of program properties such as the absence of run-time errors.

Automatic no expert knowledge required.

Semantic based on a formal modelization of the language.

Sound cover all possible executions.

Growing popularity

JavaScript #1, Python #2 on GitHub¹

¹<https://octoverse.github.com/#top-languages>

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New features

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Growing popularity

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New features

- ▶ Object orientation,
- ▶ Dynamic typing,
- ▶ Dynamic object structure,
- ▶ Introspection operators,
- ▶ `eval`.

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No standard

- ▶ CPython is the reference

⇒ manual inspection of the source code and handcrafted tests

Python's specificities

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⇒ manual inspection of the source code and handcrafted tests

Operator redefinition

- ▶ Calls, additions, attribute accesses
- ▶ Operators eventually call overloaded `__methods__`

Protected attributes

```
1 class Protected:
2     def __init__(self, priv):
3         self._priv = priv
4     def __getattr__(self, a):
5         if a[0] == "_": raise AttributeError("protected")
6         return object.__getattr__(self, a)
7
8 a = Protected(42)
9 a._priv # AttributeError raised
```

Python's specificities (II)

Dual type system

- ▶ Nominal (classes, MRO)

Fspath (from standard library)

```
1 class Path:
2     def __fspath__(self): return 42
3
4 def fspath(p):
5     if isinstance(p, (str, bytes)):
6         return p
7     elif hasattr(p, "__fspath__"):
8         r = p.__fspath__()
9         if isinstance(r, (str, bytes)):
10            return r
11        raise TypeError
12
13 fspath("/dev" if random() else Path())
```

Barrett et al. "A Monotonic Superclass Linearization for Dylan". OOPSLA 1996

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Exceptions

Exceptions rather than specific values

- ▶ `1 + "a" ↪ TypeError`
- ▶ `l[len(l) + 1] ↪ IndexError`

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Combining C and Python – motivation

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- ▶ Garbage collection

Outline

- 1 Introduction
- 2 Mopsa
- 3 A Concrete Example
- 4 Concrete Multilanguage Semantics
- 5 Experimental Evaluation
- 6 Conclusion

Mopsa

A program analysis workflow

Avering numbers

```
1 def average(l):
2     m = 0
3     for i in range(len(l)):
4         m = m + l[i]
5     m = m // (i + 1)
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7
8 l = [randint(0, 20)
9     for i in range(randint(5, 10))]
10 m = average(l)
```

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```

Proved safe?

- ▶ `m // (i+1)`
- ▶ `l[i]`

Searching for a loop invariant (l. 4)

Environment abstraction

$$m \mapsto @_{\text{int}}^{\#} \quad i \mapsto @_{\text{int}}^{\#}$$

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Stateless domains: **list content**,

Environment abstraction

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Numeric abstraction (intervals)

$$m \in [0, +\infty) \quad \underline{\text{els}}(l) \in [0, 20] \quad i \in [0, +\infty)$$

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Stateless domains: list content, **list length**

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Stateless domains: list content, list length

Environment abstraction

$$m \mapsto @_{\text{int}}^{\#} \quad i \mapsto @_{\text{int}}^{\#} \quad \underline{\text{els}}(l) \mapsto @_{\text{int}}^{\#}$$

Numeric abstraction (polyhedra)

$$m \in [0, +\infty) \quad \underline{\text{els}}(l) \in [0, 20]$$
$$0 \leq i < \underline{\text{len}}(l) \quad 5 \leq \underline{\text{len}}(l) \leq 10$$

A program analysis workflow

Averaging tasks

```
1 class Task:
2     def __init__(self, weight):
3         if weight < 0: raise ValueError
4         self.weight = weight
5
6     def average(l):
7         m = 0
8         for i in range(len(l)):
9             m = m + l[i].weight
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13 l = [Task(randint(0, 20))
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```

Proved safe?

- ▶ $m // (i+1)$
- ▶ $l[i].weight$

Searching for a loop invariant (l. 4)

Stateless domains: list content, list length

Environment abstraction

$$m \mapsto @_{int}^{\#} \quad i \mapsto @_{int}^{\#} \quad \underline{\text{els}(l)} \mapsto @_{Task}^{\#}$$
$$\underline{@_{Task}^{\#} \cdot \text{weight}} \mapsto @_{int}^{\#}$$

Numeric abstraction (polyhedra)

$$m \in [0, +\infty)$$
$$0 \leq i < \underline{\text{len}(l)} \quad 5 \leq \underline{\text{len}(l)} \leq 10$$
$$0 \leq \underline{@_{Task}^{\#} \cdot \text{weight}} \leq 20$$

Attributes abstraction

$$@_{Task}^{\#} \mapsto (\{\text{weight}\}, \emptyset)$$

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13 l = [Task(randint(0, 10), randint(1, 10))
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Conclusion

- ▶ Different domains depending on the precision
- ▶ Use of auxiliary variables (underlined)

Searching for a loop invariant (l. 4)

Stateless domains: list content, list length

Environment abstraction

$m \mapsto @^{\#} \dots i \mapsto @^{\#} \dots @^{\#} \text{Task}$

$0 \leq i < \underline{\text{len}(l)} \quad 5 \leq \underline{\text{len}(l)} \leq 10$

$0 \leq \underline{@^{\#}_{\text{Task}} \cdot \text{weight}} \leq 20$

Attributes abstraction

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Proved safe?

- ▶ $m // (i+1)$
- ▶ $l[i].weight$



Modular Open Platform for Static Analysis²

²Journault, Miné, Monat, and Ouadjaout. “Combinations of reusable abstract domains for a multilingual static analyzer”. VSTTE 2019.



Modular Open Platform for Static Analysis²

- ▶ One AST to analyze them all
 - 🚩 Multilanguage support
 - 📄 Expressiveness
 - ♻️ Reusability

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 - ▶  Semantic rewriting
 - ▶  Loose coupling
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 - ▶  Observability
- ▶ DAG of abstract domains
 - ▶  Composition
 - ▶  Cooperation

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Dynamic, semantic iterators with delegation

Universal.Iterators.Loops

Matches `while(...){...}`

Computes fixpoint using widening

Dynamic, semantic iterators with delegation

```
for(init; cond; incr) body
```

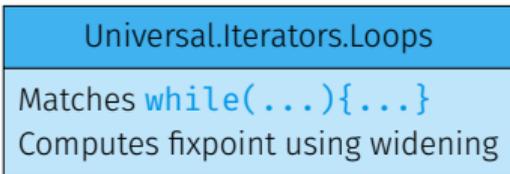
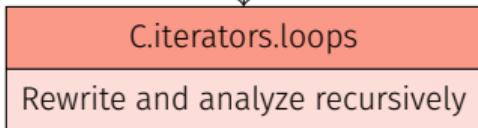
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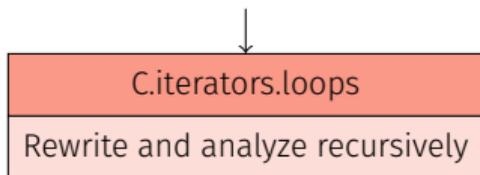
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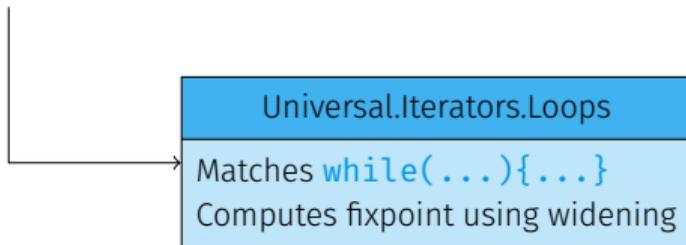


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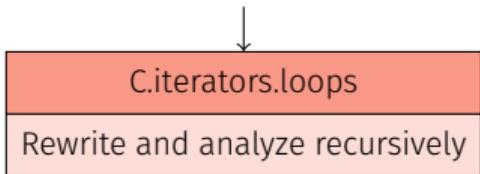
```
init;  
while(cond) {  
  body;  
  incr;  
}  
clean init
```



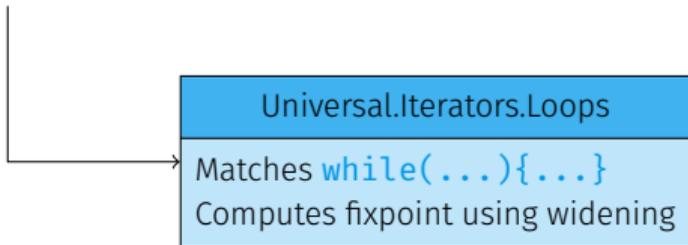
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for target in iterable: body
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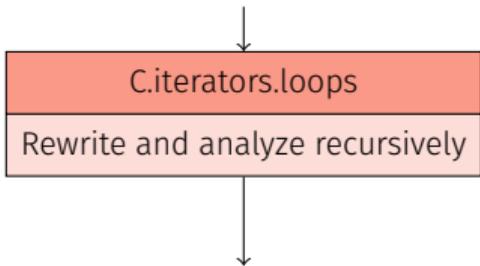


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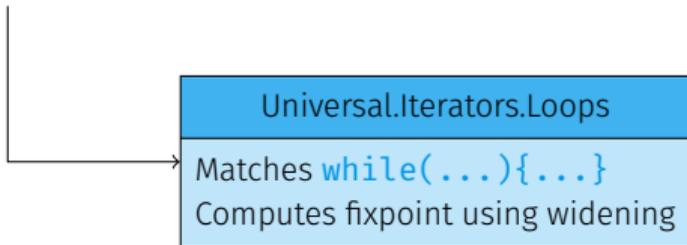


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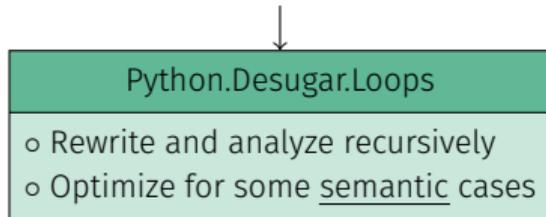
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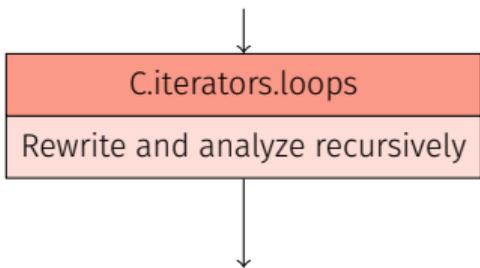


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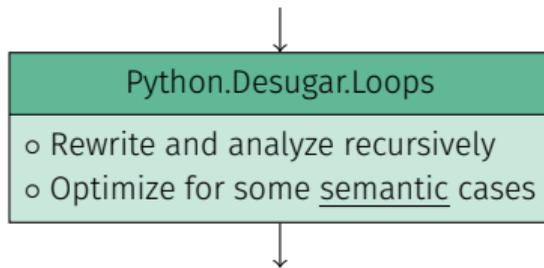
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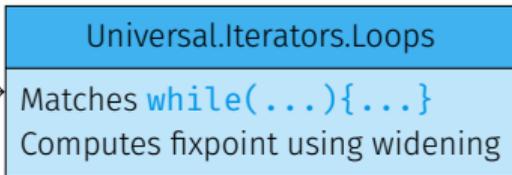


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clean init
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for target in iterable: body
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```
it = iter(iterable)  
while(1) {  
    try: target = next(it)  
    except StopIteration: break  
    body  
}  
clean it
```



A Concrete Example

Combining C and Python – example

counter.c

```
1 typedef struct {
2     PyObject_HEAD;
3     int count;
4 } Counter;
5
6 static PyObject*
7 CounterIncr(Counter *self, PyObject *args)
8 {
9     int i = 1;
10    if(!PyArg_ParseTuple(args, "|i", &i))
11        return NULL;
12
13    self->count += i;
14    Py_RETURN_NONE;
15 }
16
17 static PyObject*
18 CounterGet(Counter *self)
19 {
20     return Py_BuildValue("i", self->count);
21 }
```

count.py

```
1 from counter import Counter
2 from random import randrange
3
4 c = Counter()
5 power = randrange(128)
6 c.incr(2**power-1)
7 c.incr()
8 r = c.get()
```

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- ▶ $32 \leq \text{power} \leq 64$: OverflowError:
signed integer is greater than maximum
- ▶ $\text{power} \geq 64$: OverflowError:
Python int too large to convert to C long

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- ▶ $\text{power} = 31 \Rightarrow r = -2^{31}$
- ▶ $32 \leq \text{power} \leq 64$: OverflowError:
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How to analyze multilanguage programs?

Type annotations

```
class Counter:  
    def __init__(self): ...  
    def incr(self, i: int = 1): ...  
    def get(self) -> int: ...
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How to analyze multilanguage programs?

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How to analyze multilanguage programs?

Type annotations

Rewrite into Python code

```
class Counter:
    def __init__(self):
        self.count = 0
    def get(self):
        return self.count
    def incr(self, i=1):
        self.count += i
```

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Type annotations

Rewrite into Python code

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class Counter:  
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        self.count += i
```

- ▶ No integer wrap-around in Python

How to analyze multilanguage programs?

Type annotations

Rewrite into Python code

```
class Counter:  
    def __init__(self):  
        self.count = 0  
    def get(self):  
        return self.count  
    def incr(self, i=1):  
        self.count += i
```

- ▶ No integer wrap-around in Python
- ▶ Some effects can't be written in pure Python (e.g., read-only attributes)

How to analyze multilanguage programs?

Type annotations

Rewrite into Python code

Drawbacks of the current approaches

How to analyze multilanguage programs?

Type annotations

Rewrite into Python code

Drawbacks of the current approaches

- ▶ Not the real code

How to analyze multilanguage programs?

Type annotations

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- ▶ Not the real code
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- ▶ Not sound: some effects are not taken into account

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Our approach

How to analyze multilanguage programs?

Type annotations

Rewrite into Python code

Drawbacks of the current approaches

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- ▶ Analyze both the C and Python sources

How to analyze multilanguage programs?

Type annotations

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- ▶ Reuse previous analyses of C and Python

How to analyze multilanguage programs?

Type annotations

Rewrite into Python code

Drawbacks of the current approaches

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- ▶ Not automatic: manual conversion
- ▶ Not sound: some effects are not taken into account

Our approach

- ▶ Analyze both the C and Python sources
- ▶ Switch from one language to the other just as the program does
- ▶ Reuse previous analyses of C and Python
- ▶ Detect runtime errors in Python, in C, and at the boundary

Analysis result

counter.c

```
1 typedef struct {
2     PyObject_HEAD;
3     int count;
4 } Counter;
5
6 static PyObject*
7 CounterIncr(Counter *self, PyObject *args)
8 {
9     int i = 1;
10    if(!PyArg_ParseTuple(args, "|i", &i))
11        return NULL;
12
13    self->count += i;
14    Py_RETURN_NONE;
15 }
16
17 static PyObject*
18 CounterGet(Counter *self)
19 {
20     return Py_BuildValue("i", self->count);
21 }
```

count.py

```
1 from counter import Counter
2 from random import randrange
3
4 c = Counter()
5 power = randrange(128)
6 c.incr(2**power-1)
7 c.incr()
8 r = c.get()
```

Analysis result

| counter.c | count.py |
|--|---|
| <pre>1 typedef struct { 2 PyObject_HEAD; 3 int count; 4 } Counter; 5 6 static PyObject* 7 CounterIncr(Counter *self, 8 { 9 int i = 1; 10 if(!PyArg_ParseTuple(a 11 return NULL; 12 13 self->count += i; 14 Py_RETURN_NONE; 15 } 16 17 static PyObject* 18 CounterGet(Counter *self) 19 { 20 return Py_BuildValue(" 21 }</pre> | <pre>1 from counter import Counter 2 from random import randrange</pre> <p>△ Check #430: ./counter.c: In function 'CounterIncr': ./counter.c:13.2-18: warning: Integer overflow</p> <pre>13: self->count += i; ^^^^^^^^^^^^^^^^^</pre> <p>'(self->count + i)' has value [0,2147483648] that is larger than the range of 'signed int' = [-2147483648,2147483647] Callstack: from count.py:8.0-8: CounterIncr</p> <p>X Check #506: count.py: In function 'PyErr_SetString': count.py:6.0-14: error: OverflowError exception</p> <pre>6: c.incr(2**p-1) ^^^^^^^^^^^^^^^^^</pre> <p>Uncaught Python exception: OverflowError: signed integer is greater than maximum Uncaught Python exception: OverflowError: Python int too large to convert to C long Callstack: from ./counter.c:17.6-38::convert_single[0]: PyTuple_int from count.py:7.0-14: CounterIncr +1 other callstack</p> |

Concrete Multilanguage Semantics

Concrete definition

- ▶ Builds upon the Python and C semantics

Concrete definition

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- ▶ Defines the API: calls between languages, value conversions

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Limitations

Concrete definition

- ▶ Builds upon the Python and C semantics
- ▶ Defines the API: calls between languages, value conversions
- ▶ Shared heap, with disjoint, complementary views
- ▶ Boundary functions when objects switch views for the first time

Limitations

- ▶ Garbage collection not handled

Concrete definition

- ▶ Builds upon the Python and C semantics
- ▶ Defines the API: calls between languages, value conversions
- ▶ Shared heap, with disjoint, complementary views
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Limitations

- ▶ Garbage collection not handled
- ▶ C access to Python objects only through the API (verified by Mopsa)

Concrete definition

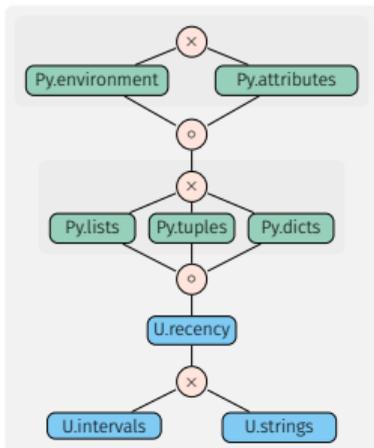
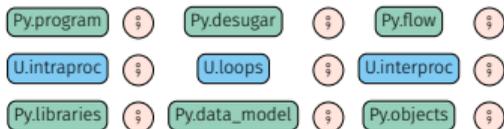
- ▶ Builds upon the Python and C semantics
- ▶ Defines the API: calls between languages, value conversions
- ▶ Shared heap, with disjoint, complementary views
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Limitations

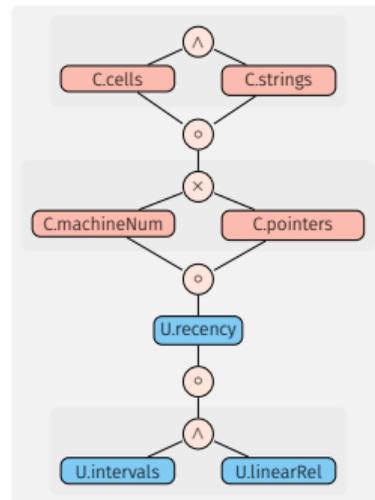
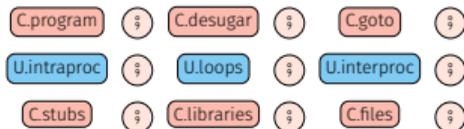
- ▶ Garbage collection not handled
- ▶ C access to Python objects only through the API (verified by Mopsa)
- ▶ Manual modelization from CPython's source code

Experimental Evaluation

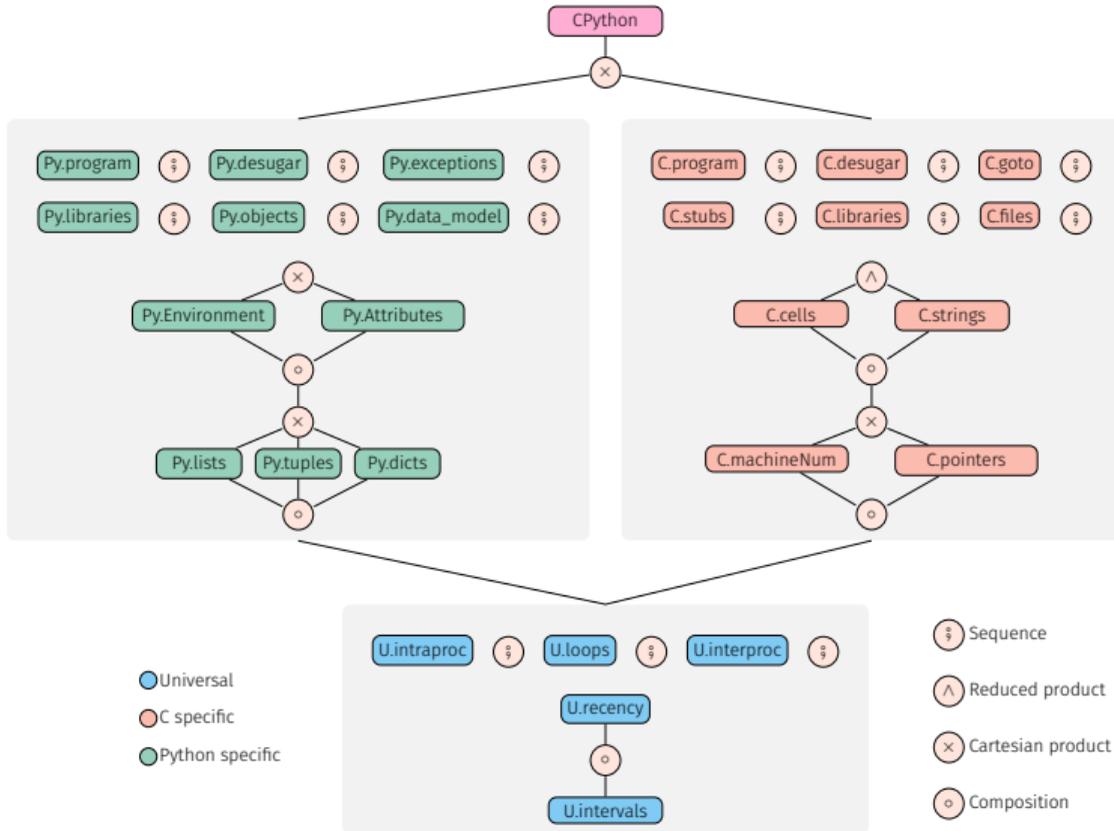
From distinct Python and C analyses...



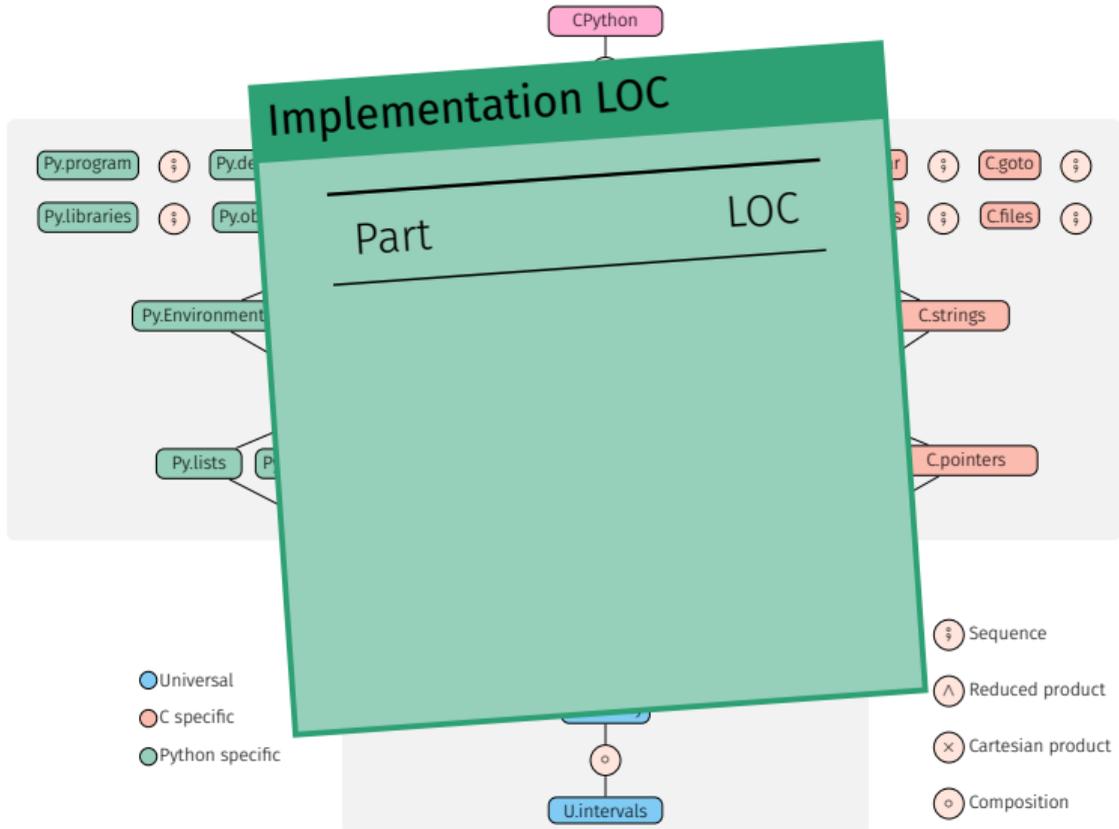
- Universal
- C specific
- Python specific
- Sequence
- ∧ Reduced product
- × Cartesian product
- Composition



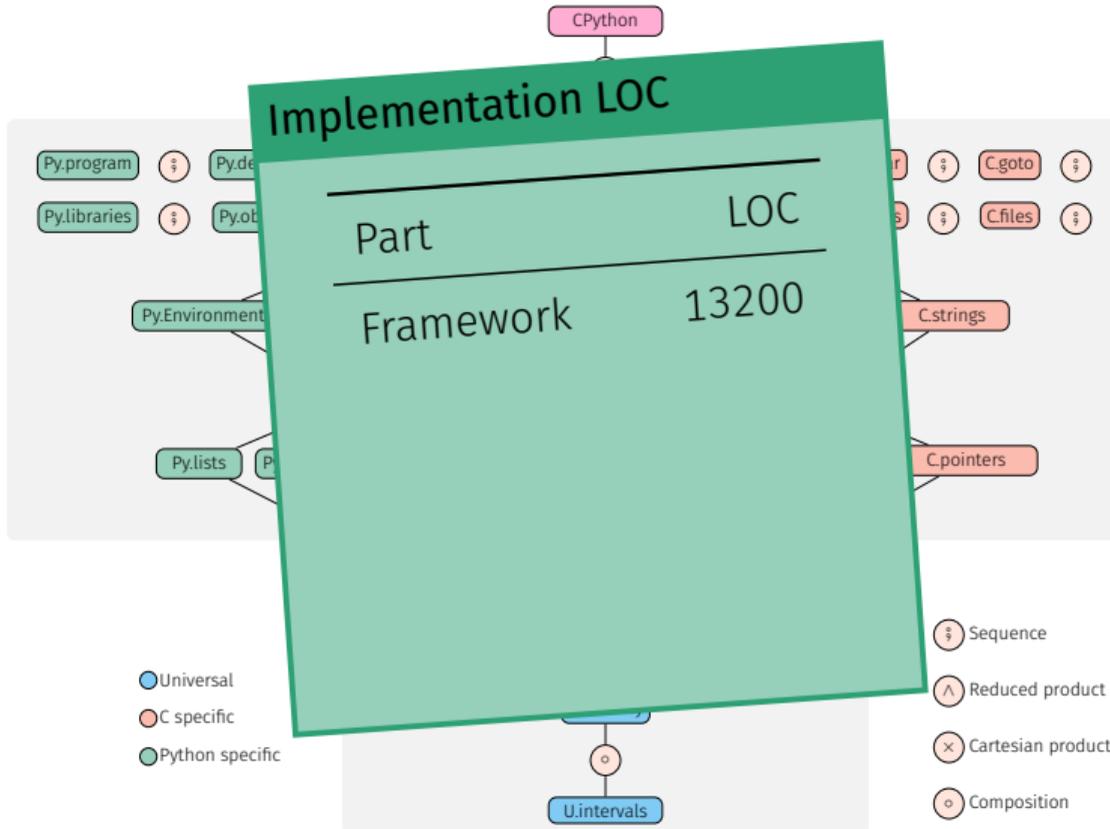
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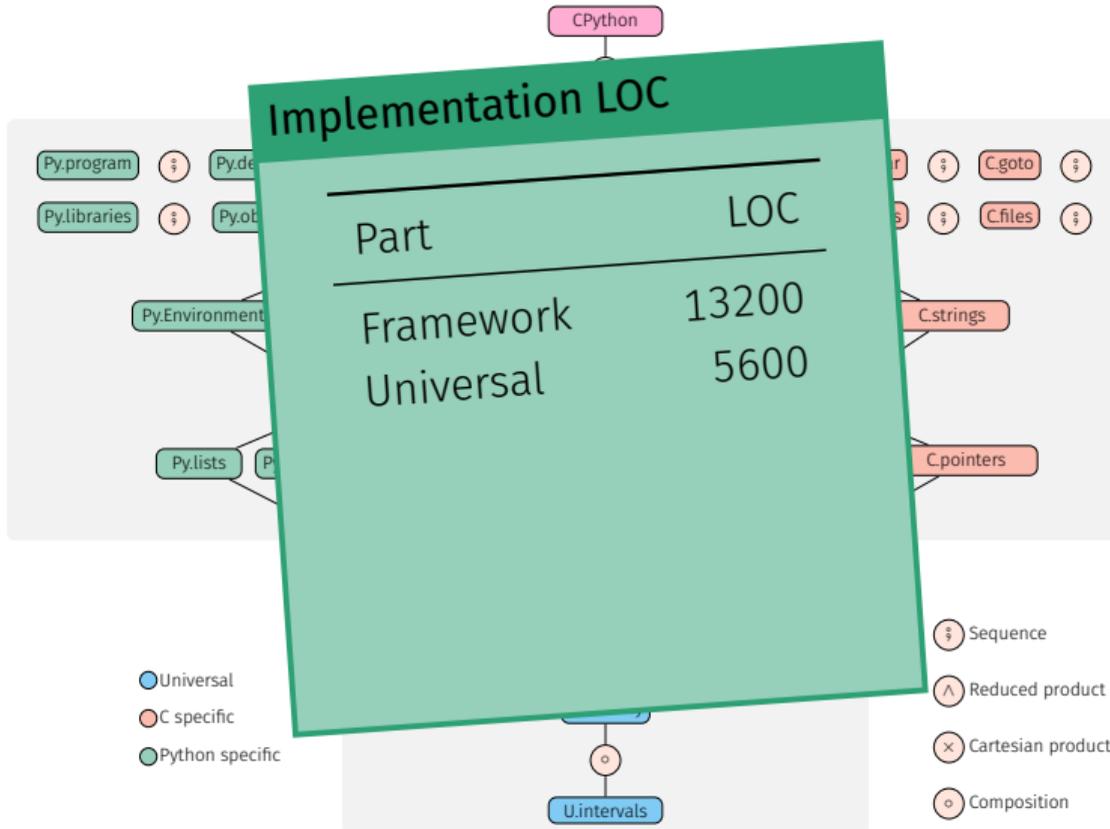
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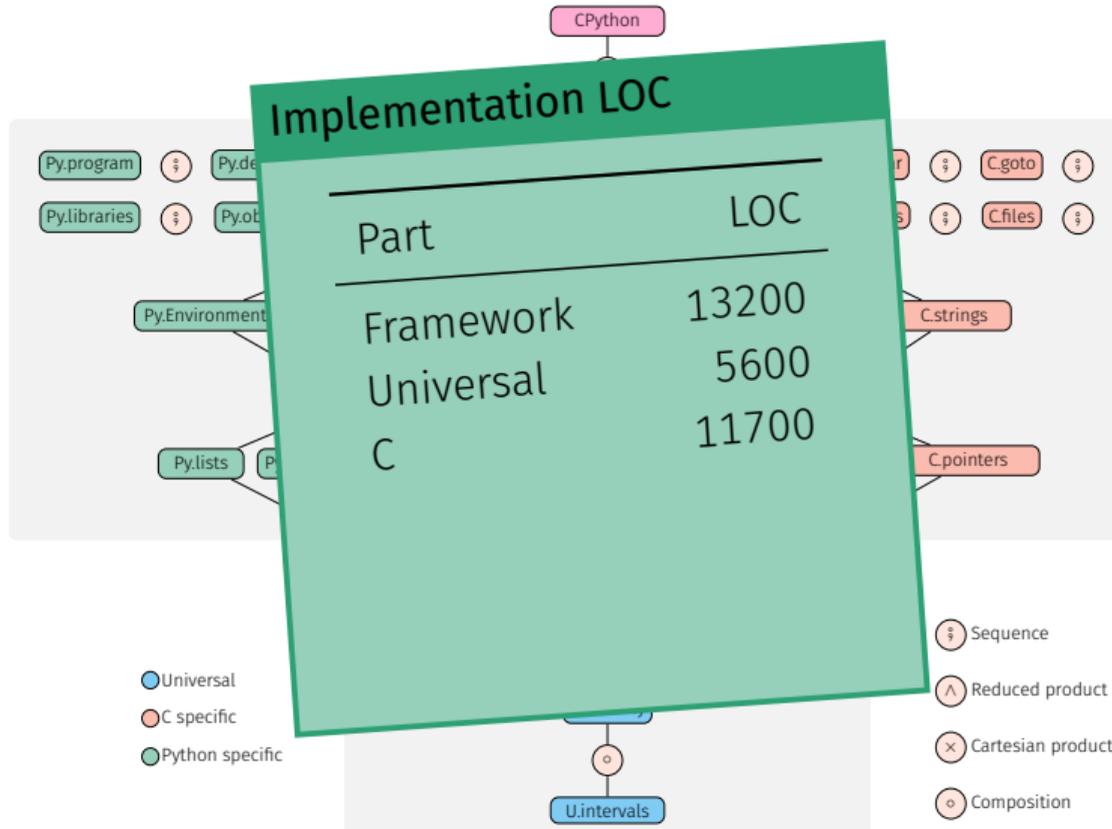
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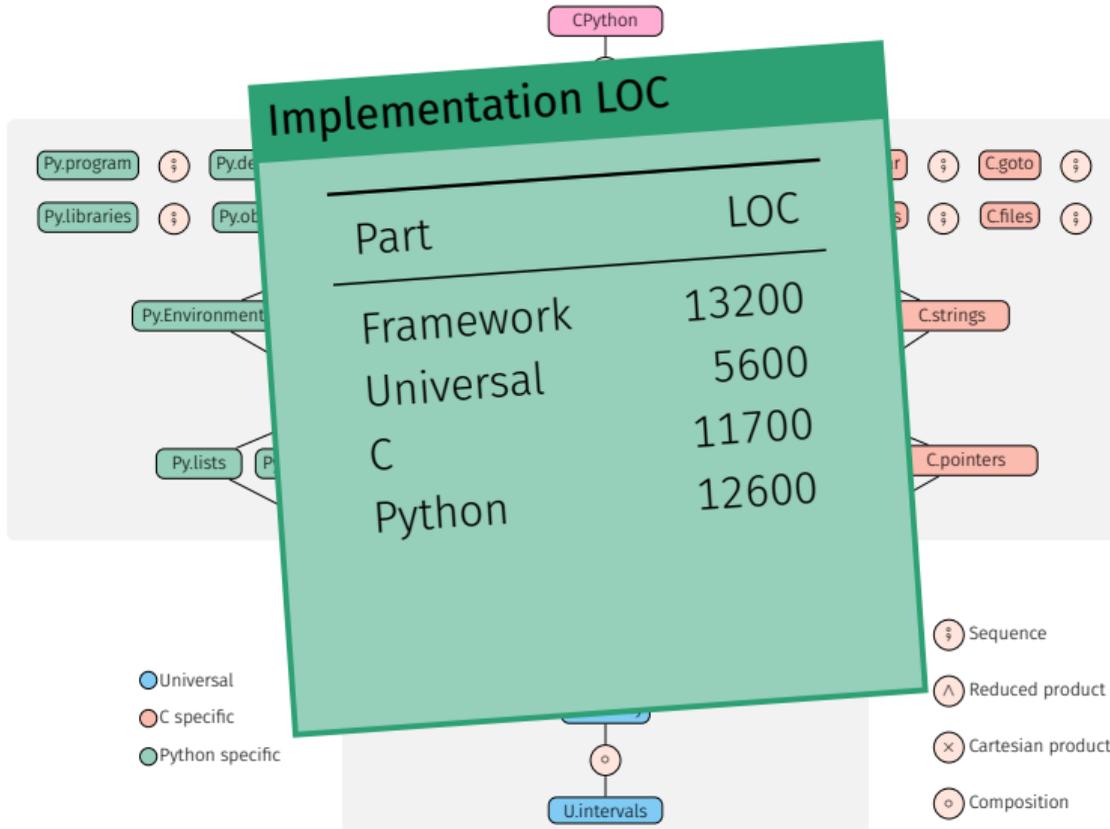
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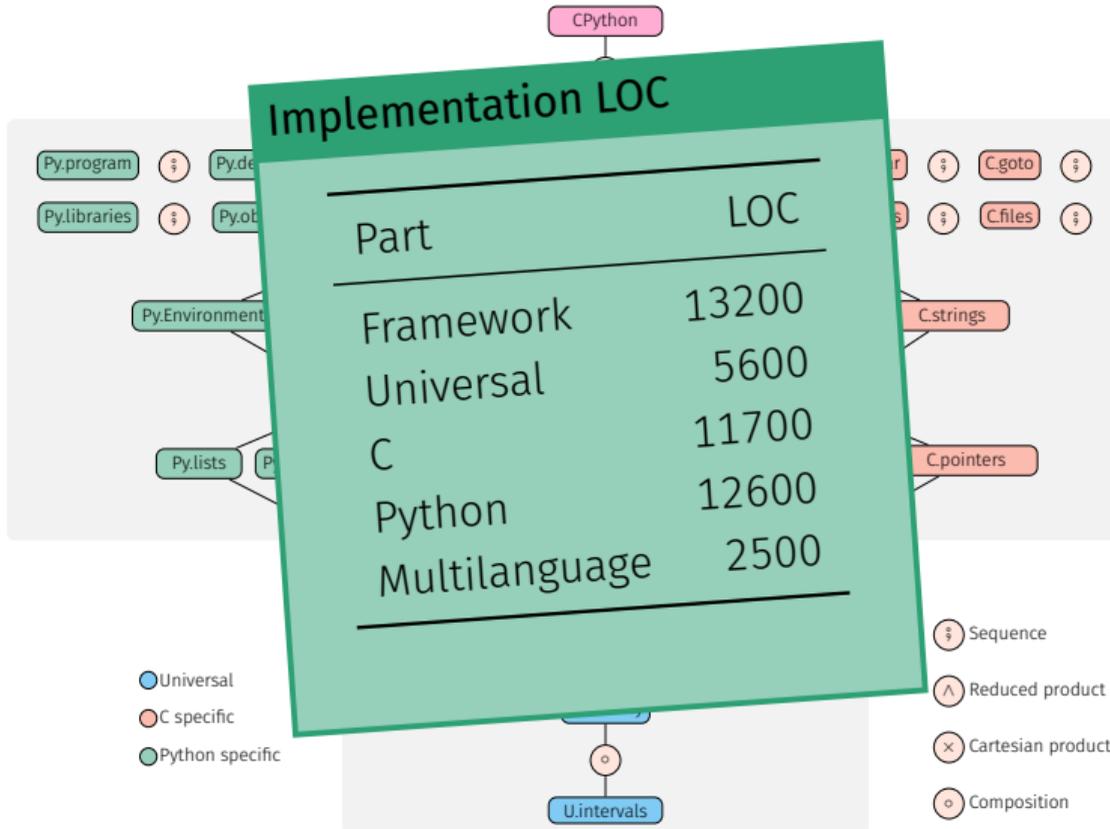
... to a multilanguage analysis!



... to a multilanguage analysis!



... to a multilanguage analysis!



Corpus selection

- ▶ Popular, real-world libraries available on GitHub, averaging 412 stars.
- ▶ Whole-program analysis: we use the tests provided by the libraries.

| Library | C | Py | Tests | 🕒 | 🔴 | 🟢 | Assertions | Py ↔ C |
|-------------|------|------|---------|------|--------------|---------------|------------|--------|
| noise | 722 | 675 | 15/15 | 18s | 99.6% (4952) | 100.0% (1738) | 0/21 | 6.5 |
| ahocorasick | 3541 | 1336 | 46/92 | 54s | 93.1% (1785) | 98.0% (4937) | 30/88 | 5.4 |
| levenshtein | 5441 | 357 | 17/17 | 1.5m | 79.9% (3106) | 93.2% (1719) | 0/38 | 2.7 |
| cdistance | 1433 | 912 | 28/28 | 1.9m | 95.3% (1832) | 98.3% (11884) | 88/207 | 8.7 |
| l1ist | 2829 | 1686 | 167/194 | 4.2m | 99.0% (5311) | 98.8% (30944) | 235/691 | 51.7 |
| bitarray | 3244 | 2597 | 159/216 | 4.2m | 96.3% (4496) | 94.6% (21070) | 100/378 | 14.8 |

$\frac{\text{safe C checks}}{\text{total C checks}}\%$

total C checks

average # transitions
between Python and C
per test

Theoretical frameworks

- ▶ Matthews and Findler³ boundary functions as value conversions between two languages.
- ▶ Buro, Crole, and Mastroeni⁴ generic framework for combining analyses of different languages.

³Matthews and Findler. “Operational semantics for multi-language programs”. 2009.

⁴Buro, Crole, and Mastroeni. “On Multi-language Abstraction - Towards a Static Analysis of Multi-language Programs”. SAS 2020.

Around the Java Native Interface (JNI)

Static translation of some of C's effects, injected back into the Java analysis.

- ▶ Effects of C code on Java heap modeled using JVML⁵
- ▶ Type inference of Java objects in C code⁶
- ▶ Extraction of C callbacks to Java⁷

- ▶ Modular analyses
- ▶ No numeric information
- ▶ Missing C runtime errors

⁵Tan and Morrisett. "Ilea: inter-language analysis across Java and C". OOPSLA 2007.

⁶Furr and Foster. "Checking type safety of foreign function calls". 2008.

⁷Lee, Lee, and Ryu. "Broadening Horizons of Multilingual Static Analysis: Semantic Summary Extraction from C Code for JNI Program Analysis". ASE 2020.

Conclusion

Difficulties

- ▶ Concrete semantics
- ▶ Memory interaction

Monat, Ouadjaout, and Miné. “A Multilanguage Static Analysis of Python Programs with Native C Extensions”. SAS 2021

Contribution: multilanguage Python/C analysis

Difficulties

- ▶ Concrete semantics
- ▶ Memory interaction

Previous works

- ▶ Type/exceptions analyses for the JNI
- ▶ No detection of runtime errors in C

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Contribution: multilanguage Python/C analysis

Difficulties

- ▶ Concrete semantics
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Previous works

- ▶ Type/exceptions analyses for the JNI
- ▶ No detection of runtime errors in C

Results

- ▶ Careful separation of the states and modelization of the API
- ▶ Lightweight domain on top of off-the-shelf C and Python analyses
- ▶ Shared underlying abstractions (numeric, recency)
- ▶ Scale to small, real-world libraries (using client code)

Monat, Ouadjaout, and Miné. “A Multilanguage Static Analysis of Python Programs with Native C Extensions”. SAS 2021

Multilanguage analyses

- ▶ Other interoperability frameworks (Cffi, Swig, Cython)
- ▶ Bigger applications

Multilanguage analyses

- ▶ Other interoperability frameworks (Cffi, Swig, Cython)
- ▶ Bigger applications

Library analyses

- ▶ Library analysis without client code
- ▶ Infer Typedshed's annotations

A Multilanguage Static Analysis of Python/C Programs with Mopsa

Questions

Raphaël Monat, Abdelraouf Ouadjaout, Antoine Miné

Binsec Seminar
26 November 2021

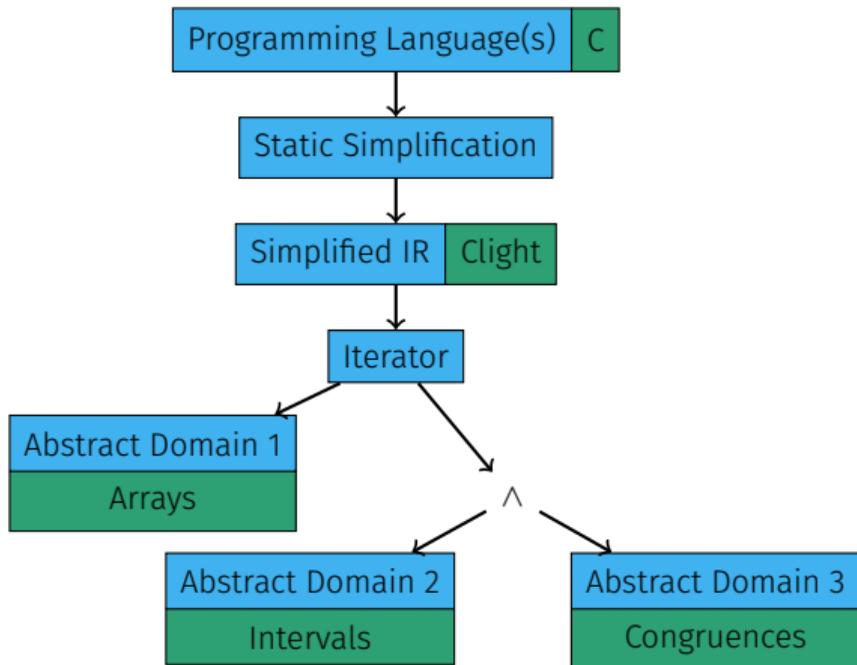
rmonat.fr



Table of backups

- 1 Table of backups
- 2 Usual implementations
- 3 Analysis of the multilanguage example
- 4 Example of recency abstraction

Usual implementations



Static simplification

- ▶ Infer's SIL IR for Java/C/C++
- ▶ May reduce the precision

Tree of abstract domains

No sharing (e.g., $\underline{\text{len}}(l)$, $\underline{\text{els}}(l)$, i)

Layered signatures

Different signatures throughout the abstract tree, and the iterator

Analysis of the multilanguage example

counter.c

```
1 typedef struct {
2     PyObject_HEAD;
3     int count;
4 } Counter;
5
6 static PyObject*
7 CounterIncr(Counter *self, PyObject *args)
8 {
9     int i = 1;
10    if(!PyArg_ParseTuple(args, "|i", &i))
11        return NULL;
12
13    self->count += i;
14    Py_RETURN_NONE;
15 }
16
17 static PyObject*
18 CounterGet(Counter *self)
19 {
```

$E^{\#}$
Py.call [Counter()] $\sigma^{\#}$

C

Pointers

```
<CounterCls,8,ptr> : {PyType_Type}
<CounterCls,232,ptr> : {Counter_methods}
```

count.py

```
1 from counter import Counter
2 from random import randrange
3
4 c = Counter()
5 power = randrange(128)
6 c.incr(2**power-1)
7 c.incr()
8 r = c.get()
```

Universal

Heap (Recency)
@CounterCls @CounterIncr
@CounterGet
Intervals

Python

Attributes
@CounterCls \mapsto {get, incr}

Environment
Counter \mapsto {@CounterCls}
@CounterCls.get \mapsto
{@c function CounterGet}
@CounterCls.incr \mapsto
{@c function CounterIncr}

Analysis of the multilanguage example

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$E_{C,call} [tp_new_wrapper(type, tuple(Counter), NULL)] \sigma^\#$

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15 }
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18 CounterGet(Counter *self)
19 {
```

$E^{\#}_{C.call} [\text{PyType_GenericNew}(\text{CounterCls}, \text{NULL}, \text{NULL})] \sigma^{\#}$

C

Pointers

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<CounterCls,8,ptr> : {PyType_Type}
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```

$E^{\#}_{C.cells} \llbracket @I\{CounterCls\} \rightarrow ob_type = CounterCls \rrbracket \sigma^{\#}$

C

Pointers

```
<CounterCls,8,ptr> : {PyType_Type}
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```

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```

E#
C.cells [@I{CounterCls}->count = 0]σ#

C

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```

Universal

Heap (Recency)

```
@CounterCls @CounterIncr
@CounterGet @I{CounterCls}
```

Intervals

```
<@I{CounterCls},16,s32> → [0, 0]
```

Python

Attributes

```
@CounterCls → {get, incr}
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```
Counter → {@CounterCls}
```

```
@CounterCls.get →
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```

$c \mapsto_p (@I\{CounterCls\}, \sigma^{\#})$

Universal

Heap (Recency)

```
@CounterCls @CounterIncr
@CounterGet @I{CounterCls}
```

Intervals

```
<@I{CounterCls},16,s32>  $\mapsto$  [0, 0]
```

count.py

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Python

Attributes

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@I{CounterCls}  $\mapsto$   $\emptyset$ 
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Counter  $\mapsto$  {@CounterCls}
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```

S#
Py.env [c = @I{CounterCls}] σ#

C

Pointers

```
<CounterCls,8,ptr> : {PyType_Type}
<CounterCls,232,ptr> : {Counter_methods}
<@I{CounterCls},8,ptr> : {CounterCls}
```

Universal

Heap (Recency)

```
@CounterCls @CounterIncr
@CounterGet @I{CounterCls}
```

Intervals

```
<@I{CounterCls},16,s32> → [0, 0]
```

Python

Attributes

```
@CounterCls → {get, incr}
@I{CounterCls} → ∅
```

Environment

```
Counter → {@CounterCls}
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{@c function CounterGet}
@CounterCls.incr →
{@c function CounterIncr}
c → {@I{CounterCls}}
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```

$S_{Py}^{\#} [power = randrange(128)] \sigma^{\#}$

C

Pointers

$\langle CounterCls, 8, ptr \rangle : \{PyType_Type\}$
 $\langle CounterCls, 232, ptr \rangle : \{Counter_methods\}$
 $\langle @I\{CounterCls\}, 8, ptr \rangle : \{CounterCls\}$

Universal

Heap (Recency)

$@CounterCls$ $@CounterIncr$
 $@CounterGet$ $@I\{CounterCls\}$

Intervals

$\langle @I\{CounterCls\}, 16, s32 \rangle \mapsto [0, 0]$
 $power \mapsto [0, 127]$

Python

Attributes

$@CounterCls \mapsto \{get, incr\}$
 $@I\{CounterCls\} \mapsto \emptyset$

Environment

$Counter \mapsto \{@CounterCls\}$
 $@CounterCls.get \mapsto$
 $\{@c \text{ function } CounterGet\}$
 $@CounterCls.incr \mapsto$
 $\{@c \text{ function } CounterIncr\}$
 $c \mapsto \{@I\{CounterCls\}\}$
 $power \mapsto \{@I\{int\}\}$

Example of recency abstraction

```
1 class Task:
2     def __init__(self, weight):
3         if weight < 0: raise ValueError
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{@#(Task, r)

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$\{ \text{c}\#(\text{Task}, r) \cdot \text{weight} \mapsto [2, 2] \}$

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$\left\{ \begin{array}{l} @^\#(\text{Task}, r) \\ @^\#(\text{Task}, o) \cdot \text{weight} \mapsto [2, 2] \end{array} \right.$

Allocation: $@^\#(\text{Task}, r) \rightsquigarrow @^\#(\text{Task}, o)$

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$$\left\{ \begin{array}{l} @^\#(\text{Task}, r) \cdot \text{weight} \mapsto [5, 5] \\ @^\#(\text{Task}, o) \cdot \text{weight} \mapsto [1, 4] \end{array} \right.$$