Formalizing Date Arithmetic and Statically Detecting Ambiguities for the Law

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Legal implementations

Some legal implementations are critical software: taxes, benefits

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Catala

- a DSL for computational laws

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- a DSL for computational laws
- providing transparency
- easing maintenance
- through interdisciplinary work

Computing dates

```
$ date -d "2024-01-31 + 1 month" +%F
```

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$ date -d "2024-01-31 + 1 month" +%F
2024-03-02
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Non-monotonic behavior?!

A wide variety of date semantics

Different legal bodies and choices

- 1 month = 30 days (Council of European Communities)

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$\Longrightarrow$ Formal, flexible semantics required!

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$\Longrightarrow$ Formal, flexible semantics required! Focus on Gregorian calendar.

Outline

Semantics

Semantics – Values

$$
\begin{array}{llll}
\text{values} & v & ::= & (y, m, d) \mid \bot \\
\text{date unit} & \delta & ::= & y \mid m \mid d \\
\text{expressions} & e & ::= & v \mid e \ +_{\delta} \ n
\end{array}
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$$
\text{nb\_days}(y, m) = \begin{cases} 29 \text{ if } m = 2 \land \text{is\_leap}(y) \\ 28 \text{ if } m = 2 \land \neg\text{is\_leap}(y) \\ 30 \text{ if } m \in \{\, \texttt{Apr}, \texttt{Jun}, \texttt{Sep}, \texttt{Nov} \,\} \\ 31 \text{ otherwise} \end{cases}
$$

Semantics – invalid dates

Invalid initial dates propagate errors

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$$\frac{d < 1}{(y, m, d) +_d n \to \bot}\;\textsc{Add-Days-Err1}$$

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$$\frac{d < 1}{(y, m, d) +_d n \to \bot}\ \textsc{Add-Days-Err1} \qquad \frac{d > \text{nb\_days}(y, m)}{(y, m, d) +_d n \to \bot}\ \textsc{Add-Days-Err2}$$

Semantics – some cases of month addition

$$\textsc{Add-Month}\quad \frac{1 \leq m + n \leq 12}{(y, m, d) +_m n \rightarrow (y, m + n, d)}$$

Semantics – some cases of month addition

$$\frac{1 \leq m + n \leq 12}{(y, m, d) +_m n \rightarrow (y, m + n, d)} \quad \textsc{Add-Month}$$

$$\frac{m + n > 12}{(y, m, d) +_m n \rightarrow (y + 1, m, d) +_m (n - 12)} \quad \textsc{Add-Month-Over}$$

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$$\textsc{Add-Month-Over}\quad \frac{m + n > 12}{(y, m, d) +_m n \rightarrow (y + 1, m, d) +_m (n - 12)}$$

Similar cases for ADD-MONTH-UNDER, year, day addition.

Semantics – Rounding

$$(2024, 01, 31) +_m 1 \rightarrow (2024, 02, 31)$$

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Rounding to valid dates required!

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Rounding to valid dates required!

$$\begin{array}{llll} \text{rounding mode} & r & ::= & \uparrow \mid \downarrow \mid \bot \\ \text{expressions} & e & ::= & v \mid e +_\delta n \mid \mathrm{rnd}_r\ e \end{array}$$

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$$\text{rnd}_\uparrow(2024, 02, 31) = (2024, 03, 01)$$

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$$(2024, 01, 31) +_m 1 \to (2024, 02, 31)$$

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rounding mode	r	$::=$	$\uparrow \mid \downarrow \mid \bot$
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Coreutils-like rounding not defined here

Semantics – Rounding

$$\frac{1 \leq d \leq \text{nb\_days}(y, m)}{\text{rnd}_r(y, m, d) \rightarrow (y, m, d)}\ \textsc{Round-Noop}$$

Semantics – Rounding

Round-Noop

$$\frac{1 \leq d \leq \text{nb\_days}(y, m)}{\text{rnd}_r(y, m, d) \rightarrow (y, m, d)}$$

Round-Down

$$\frac{d > \text{nb\_days}(y, m)}{\text{rnd}_{\downarrow}(y, m, d) \rightarrow (y, m, \text{nb\_days}(y, m))}$$

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Round-Up

$$\frac{d > \text{nb\_days}(y, m) \qquad (y, m, d) +_m 1 \xrightarrow{*} (y', m', d')}{\text{rnd}_\uparrow(y, m, d) \rightarrow (y', m', 1)}$$

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Round-Err2

$$\frac{d > \text{nb\_days}(y, m)}{\text{rnd}_\perp(y, m, d) \rightarrow \perp}$$

Date-period addition

Given a period (ys, ms, ds):

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Ambiguous expression

A date expression e is ambiguous iff $\mathrm{rnd}_\perp(e) \xrightarrow{*} \perp$

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A date expression e is ambiguous iff $\mathrm{rnd}_\bot(e) \xrightarrow{*} \bot$

iff roundings e yield different values

Formalized Properties

Non-properties

Commutativity of addition

$$(2024, 03, 31) +_{\uparrow} 1\text{m} +_{\uparrow} 1\text{d} = (2024, 05, 01) +_{\uparrow} 1\text{d} = (2024, 05, 02)$$

Non-properties

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Associativity of addition

$$(2024, 03, 31) +_\uparrow 1\text{m} +_\uparrow 1\text{m} = (2024, 05, 01) +_\uparrow 1\text{m} = (2024, 06, 01)$$

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$$(2024, 03, 31) +_{r} 2 = (2024, 05, 31)$$

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All formalized with the F* proof assistant. More in the paper & artefact.

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Well-formedness

For any date d, any period p, any value v, and $r \in \{\downarrow, \uparrow\}$, we have:

$$\text{valid}(d) \wedge d +_r p \xrightarrow{*} v \Rightarrow \text{valid}(v)$$

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Date addition is monotonic

For any dates d_1, d_2, period p, $r \in \{\downarrow, \uparrow\}$, if $d_1 < d_2$, then $d_1 +_r p \leq d_2 +_r p$

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Loose bound in conclusion of monotonicity

$(2024, 03, 30) +_\downarrow 1m = (2024, 04, 30) = (2024, 03, 31) +_\downarrow 1m$

Formalized properties (II)

Rounding is monotonic

For all date *d*, period *p*:

1. $d +_{\downarrow} p \leq d +_{\uparrow} p$
2. $d +_{\bot} p \neq \bot \Rightarrow d +_{\downarrow} p = d +_{\uparrow} p = d +_{\bot} p$

Formalized properties (II)

Rounding is monotonic

For all date d, period p:

1. $d +_{\downarrow} p \leq d +_{\uparrow} p$
2. $d +_{\perp} p \neq \perp \Rightarrow d +_{\downarrow} p = d +_{\uparrow} p = d +_{\perp} p$

Equivalence of year and month addition

For all date d, for all integer n, $d +_{y} n = d +_{m} (12 * n)$.

Formalized properties (III)

Ambiguous month addition

For all valid date d, integer n such that $d +_m n \xrightarrow{*} (y, m, day)$:

$$\text{nb\_days}(y, m) < day \Leftrightarrow \text{rnd}_\perp((y, m, day)) \xrightarrow{*} \perp$$

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$\Longrightarrow$ core result needed for our static analysis

Rounding-insensitivity Static Analysis

Meaningful ambiguities

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d + 1 month <= April 15 2024
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To reduce the need for costly legal interpretations

Rounding-insensitivity Static Analysis

Abstracting dates in a fixed rounding mode

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$\mathsf{d}(d_1) \in [1, 31] \wedge \mathsf{m}(d_1) \in [1, 12] \wedge \mathsf{y}(d_1) = 2024$: all valid dates of 2024

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$$\gamma_{\mathcal{N}} : \mathcal{N}^\sharp \to \mathcal{P}(\mathcal{V} \to \mathbb{Z})$$

$$\gamma_{\text{YMD}} : \begin{cases} \mathcal{N}^\sharp & \to & \mathcal{P}(\mathcal{V} \to \mathcal{D}) \\ n^\sharp & \mapsto & \bigcup_{\rho \in \gamma_{\mathcal{N}}(n^\sharp)} \{\, e \mid \forall v \in \text{dom}(e), e(v) = (y, m, d) \wedge \text{valid}(y, m, d) \\ & & \qquad \wedge y = \rho(\mathbf{y}(v)) \wedge m = \rho(\mathbf{m}(v)) \wedge d = \rho(\mathbf{d}(v)) \,\} \end{cases}$$

YMD domain – month addition

Goal

Given a rounding mode, compute resulting dates from $d^{\#} +^{\#}_{m} n$, where $d^{\#}$ represents a set of dates.

Soundly derived from the ambiguous addition theorem.

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Algorithm: compute resulting month, year, then 4 cases:

- No rounding,
- Rounding, 30-day month,
- Rounding, non-leap years 28 Feb,
- Rounding, leap years, 29 Feb.

Partitioning used in practice.

YMD domain – month addition (II)

```
type case = expr * state
type cases = case list
let switch abs =
  List.map (fun (cond : expr, k : state -> case) -> k (assume cond abs))

let add_months (r: rnd) ((d, m, y): var^3) (nb_m: int) (abs: state): cases =
  let res_m: expr = 1 + (m - 1 + nb_m) % 12 in
  let res_y: expr = y + (m - 1 + nb_m) / 12 in
  switch abs
  [
    mk_true,
      mk_date d res_m res_y;
    d > 30 && is_one_of res_m [Apr;Jun;Sep;Nov],
      round r 30 res_m res_y;
    d > 28 && res_m = Feb && not (is_leap res_y),
      round r 28 res_m res_y;
    d > 29 && res_m = Feb && is_leap res_y,
      round r 29 res_m res_y
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```

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`date d1 = rand_date(); date d2 = d1 + 1 month;` rounding down.

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  $\mathrm{d}(d1) = 31, \ \mathrm{m}(d1) \in \{\texttt{Mar}, \texttt{May}, \texttt{Aug}, \texttt{Oct}\}, \ \mathrm{m}(d2) = \mathrm{m}(d1) + 1, \mathrm{y}(d2) = \mathrm{y}(d1)$

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- No rounding $\mathrm{d}(d1) = \mathrm{d}(d2),\ \mathrm{m}(\mathtt{d2}) \equiv_{12} \mathrm{m}(\mathtt{d1}) + 1, \mathrm{y}(d1) \leq \mathrm{y}(d2) \leq \mathrm{y}(d1) + 1$[1]

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$$\mathrm{d}(d1) = 31,\ \underbrace{\mathrm{m}(d1) \in \{\texttt{Mar}, \texttt{May}, \texttt{Aug}, \texttt{Oct}\}}_{\text{Bounded set of ints}},\ \underbrace{\mathrm{m}(d2) = \mathrm{m}(d1) + 1, \mathrm{y}(d2) = \mathrm{y}(d1)}_{\text{Polyhedra}}$$

- No rounding $\mathrm{d}(d1) = \mathrm{d}(d2),\ \underbrace{\mathrm{m}(\mathrm{d2}) \equiv_{12} \mathrm{m}(\mathrm{d1}) + 1}_{\text{Linear congruence domain}}, \mathrm{y}(d1) \leq \mathrm{y}(d2) \leq \mathrm{y}(d1) + 1$[1]

[1]Actually, $12\,\mathrm{y}(\mathrm{d1}) + \mathrm{m}(\mathrm{d1}) \leq 12\,\mathrm{y}(\mathrm{d2}) + 11 \wedge 12\,\mathrm{y}(\mathrm{d2}) \leq 12\,\mathrm{y}(\mathrm{d1}) + \mathrm{m}(\mathrm{d1}) + 1$

Rounding-insensitivity Static Analysis

Lifting to both rounding modes

Back to rounding-insensitivity detection

- Semantics on product programs with both rounding modes.

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$$\mathbb{E}_r[\![e]\!] : \mathcal{P}(\mathcal{E}) \to \mathcal{P}(\mathsf{Val}), r \in \{\uparrow, \downarrow\} \quad \rightsquigarrow$$

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- Inspired by Delmas, Ouadjaout, and Miné. “Static Analysis of Endian Portability by Abstract Interpretation”. SAS 2021.

Abstract double semantics

Shallow variable duplication depending on their rounding mode.

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- 30-day month

$$\mathsf{d}(d1) = 31, \mathsf{m}(d1) \in \{\mathtt{Mar}, \mathtt{May}, \mathtt{Aug}, \mathtt{Sep}\}$$
$$\downarrow\mathsf{d}(d2) = 30, \downarrow\mathsf{m}(d2) \in \{\mathtt{Apr}, \mathtt{Jun}, \mathtt{Sep}, \mathtt{Nov}\}$$
$$\uparrow\mathsf{d}(d2) = 1, \uparrow\mathsf{m}(d2) \in \{\mathtt{May}, \mathtt{Jul}, \mathtt{Oct}, \mathtt{Dec}\}$$
$$\downarrow\mathsf{y}(d2) = \uparrow\mathsf{y}(d2) = \mathsf{y}(d1)$$

Implementation into Mopsa

- Open-source static analysis platform

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- C, Python, C+Python programs

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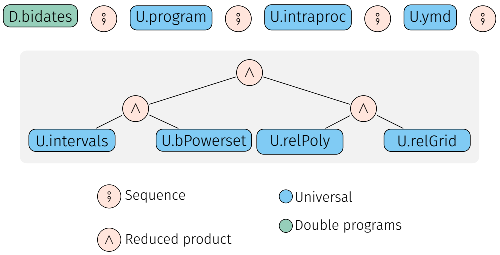

Extracted sample from French housing benefits

```
date current = rand_date();
date birthday = rand_date();
date intermediate = birthday + [2 years, 0 months, 0 days];
date limit = first_day_of(intermediate);
assert(sync(current < limit));
```

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1 date current = rand_date();
2 date birthday = rand_date();
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5 assert(sync(current < limit));
```

```
5: assert(sync(current < limit));
               ^^^^^^^^^^^^^^^

Desynchronization detected: (current < limit). Hints:
↑month(limit) = 3, ↑day(limit) = 1, ↓month(limit) = 2, ↓day(limit) = 1,
↑month(intermediate) = 3, ↑day(intermediate) = 1, ↓month(intermediate) = 2,
↓day(intermediate) = 28, month(birthday) = 2, day(birthday) = 29,
year(birthday) =[4] 0, month(current) = 2, day(current) = [1,29],
year(current) = ↑year(intermediate) = ↑year(limit)
= ↓year(intermediate) = ↓year(limit) = year(birthday) + 2
```

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```
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                ^^^^^^^^^^
Desynchronization detecte
↑month(limit) = 3, ↑day(l
↑month(intermediate) = 3,                                  ,
↓day(intermediate) = 28,
year(birthday) =[4] 0, mo
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Computed, actual counter-example

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Computed, actual counter-example

- `current` is in Feb. of year *y*

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- `current` is in Feb. of year y
- `birthday` is 29 Feb. of leap year $y - 2$

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Computed, actual counter-example

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- `intermediate` is either 28 Feb. or 1 March of y

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Case Study: French Housing Benefits

Catala, a DSL for computational laws

Article D823-20 du code de la construction réglementaire

La prime de déménagement est attribuée aux personnes ou aux ménages ayant à charge au moins trois enfants nés ou à naître et qui s'installent dans un nouveau logement ouvrant droit à l'une des aides personnelles au logement au cours d'une période comprise entre le premier jour du mois civil suivant le troisième mois de grossesse au titre d'un enfant de rang trois ou plus et le dernier jour du mois précédant celui au cours duquel cet enfant atteint son deuxième anniversaire.
Cette prime est due si le droit à l'aide est ouvert dans un délai de six mois à compter de la date d'emménagement.

````
```catala
champ d'application ÉligibilitéPrimeDeDéménagement:
 règle condition_période_déménagement sous condition
  (selon informations.date_naissance_troisième_enfant_ou_plus
  sous forme
  -- PlusDeTroisEnfants de date_naissance_ou_grossesse:
   (selon date_naissance_ou_grossesse sous forme
    -- DateDeNaissance de date_naissance:
    date_courante < (premier_jour_du_mois de (date_naissance + 2 an))
    # ...
   )
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 conséquence rempli
```
````

Merigoux, Chataing, and Protzenko. "Catala: a programming language for the law". 2021
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Catala, a DSL for computational laws

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````

- Literate programming
- Lawyer-developer duos
- Default logic tailored to the law

Merigoux, Chataing, and Protzenko. "Catala: a programming language for the law". 2021
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Contributions to Catala

- Date-rounding library `dates-calc`

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Case Study – Catala for the French Housing Benefits

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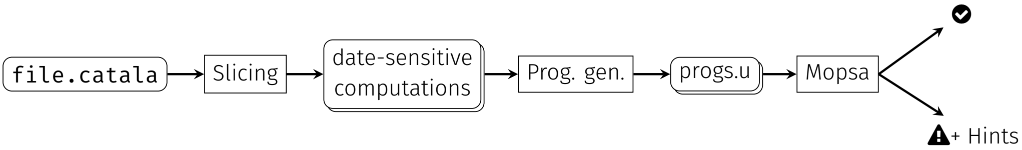
French Housing Benefits

20,000 Loc of Catala code (including text spec.)

Date ambiguity detection pipeline



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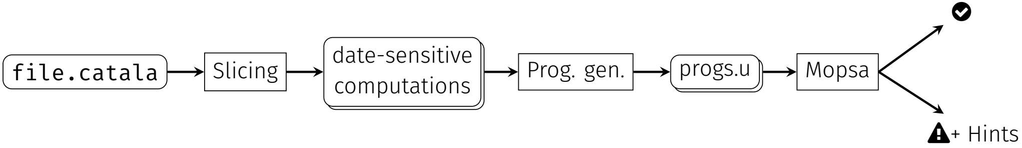

2 rounding-sensitive cases detected

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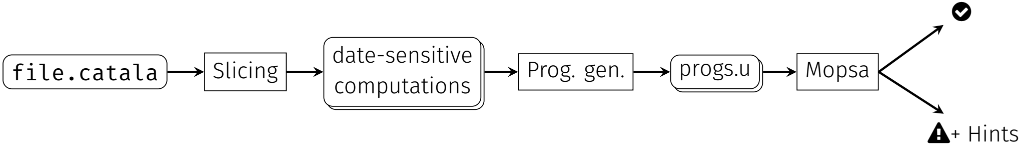

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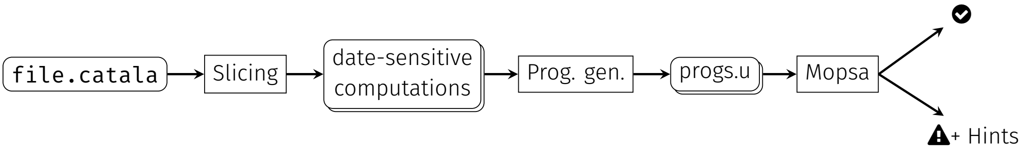
Intra-scope extraction for now

Date ambiguity detection pipeline



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Intra-scope extraction for now

Manual inter-scope extraction

16 additional cases:

- 10 can be proved safe (assuming `current_date` ≥ `2023`)
- Other are real issues

Conclusion

Related Work

Survey of implementations

- Java, `boost` round down
- Python `stdlib`: no month addition
- Inconsistency in spreadsheets

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Floating-point arithmetic

- FP widely used & more complex!
- Different rounding modes
- No analysis of rounding-sensitivity?

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"Automatic Verification of Catala programs" project, funded by Inria

Formalizing Date Arithmetic and Statically Detecting Ambiguities for the Law

Questions

Raphaël Monat, Aymeric Fromherz, Denis Merigoux

`rmonat.fr`

Épicure seminar
23 February 2024

Inria

Université de Lille