Formal methods meet legal implementations

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Other Research Interests in SyCoMoRES Team

- Scheduling for real-time embedded systems
- Binary code analysis [Bal+19] (for worst-case execution time, security)
- Type systems for privacy, Kleene algebras

Legal implementations?

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Need more?

See “Automating public services: learning from cancelled systems” [Red+22].

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  - "participants (esp. CS) expressed unwarranted confidence in [tool utility]"

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$\simeq$ yearly legislative changes needing implementations

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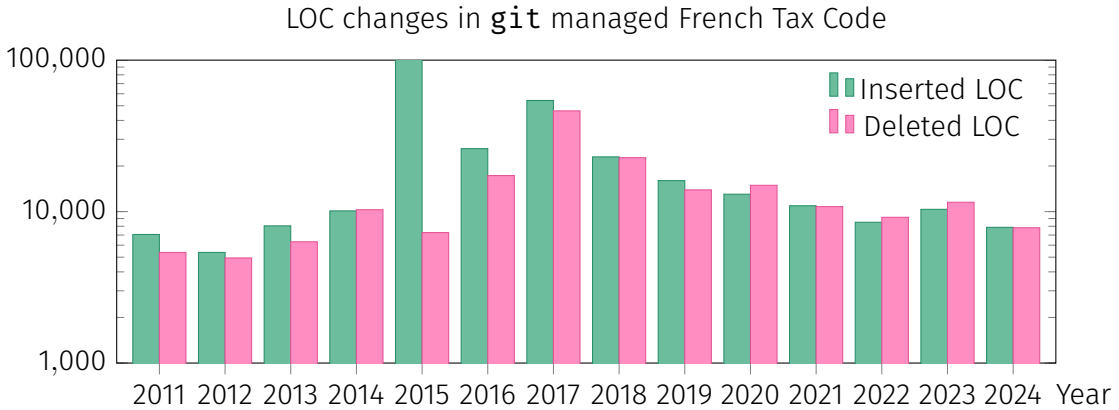

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French Benefits

- 1996 Centralization of multiple benefits
- Modernization efforts, types of benefits has tripled

Towards high-assurance legal implementations

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- Reproducibility of the computation?
- Compliance with the law, acting as specification?
- Usability by public administrations, citizens, NGOs, researchers?
- Accurate simulations?

Outline

Joint work

- Marie Alauzen
- Justine Banuls
- Vincent Botbol
- Nicolas Chataing
- Caroline Flori
- Aymeric Fromherz
- Louis Gesbert
- Pierre Goutagny (slides ack)
- Estelle Hary
- Liane Huttner
- Sarah Lawsky
- Denis Merigoux (now transfer lead)
- Romain Primet
- Jonathan Protzenko
- Émile Rolley
- Lilya Slimani

A Modern Compiler for the French Tax Code [MMP21]

Legal implementations

French income tax

- 38M households, 75Md€ of income
- Made public in April 2016 : $\simeq$ 92kLoc M, custom language
- ⚠ Computation not reproducible in 2019

Example M code

Variable declaration

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IRNETBIS : computed primrest = 0 : "net income tax before 8ZI hack";
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Computation rule

```
rule 221220:
application : iliad ;
IRNETBIS = max(0, IRNETTER -
                  PIR * positive(THRESHOLD_12 - IRNETTER + PIR)
                      * positive(THRESHOLD_12 - PIR)
                      * positive_or_zero(IRNETTER - SEUIL_12));
```

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- Use of floating-point numbers, booleans are zero and one
- `undef` value

A formal semantics for M

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The `undef` value

- Used for: default inputs, runtime errors & missing cases in inline conditionals
- Fun facts: $f + \texttt{undef} = f$, $f \div 0 = 0$, $x[|x| + 1] = \texttt{undef}$, $x[-1] = 0$...

Is this the end?

Test failures

Aug 2019: only 20% of DGFiP tests passed...

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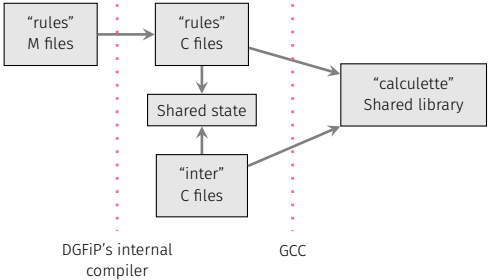
Patience is key

Aug. 2019	Sent official technical questions to DGFiP
Jan. 2020	Meeting with DGFiP (5 levels of hierarchy involved!)
Apr. 2020	Agreement signed
Jun. 2020	First access to the unpublished sources!

DGFiP's legacy architecture

After 9 months of negotiations, we're in!



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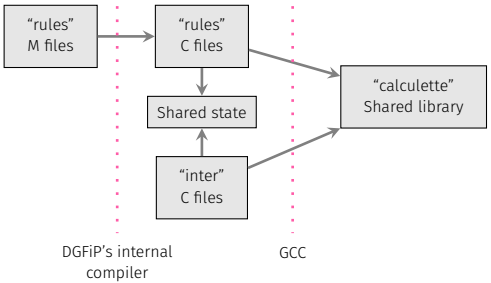

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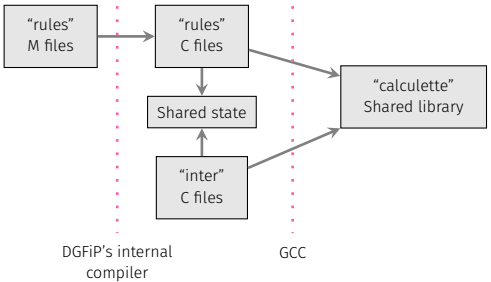

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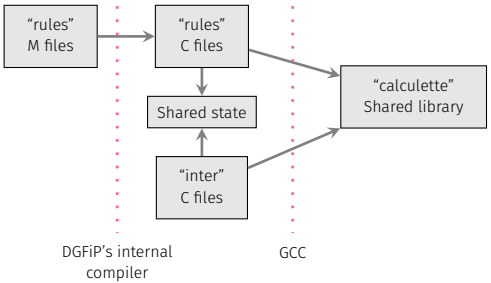

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DSLs to the rescue! Introducing M++

- High-level, no mutable state under the hood
- Tailored for the needs of the "inter" files and DGFiP devs
- 6,000 lines of "inter" C code ⇒ 100 lines of M++

MLANG's architecture

MLANG: written in OCaml, 10k lines of code

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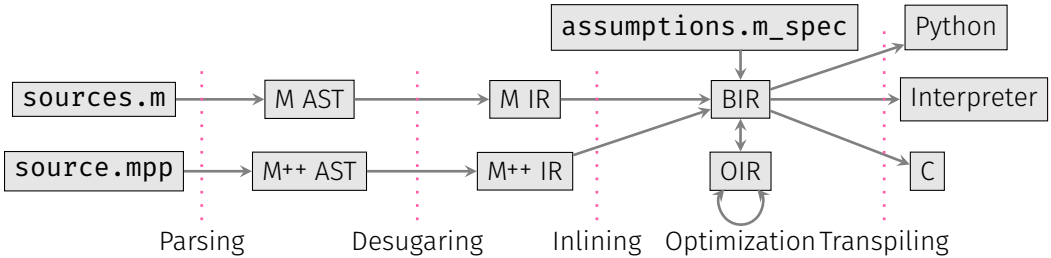

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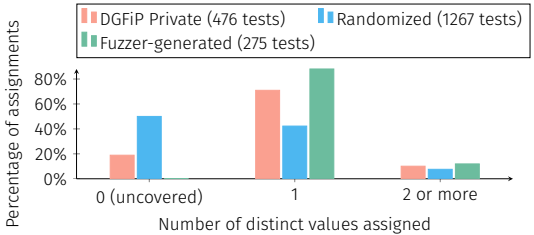

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Compiler optimizations

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Interacting with DGFiP

- Long term work: 9 months to access the missing C code
- Pedagogy required: non-technical environment

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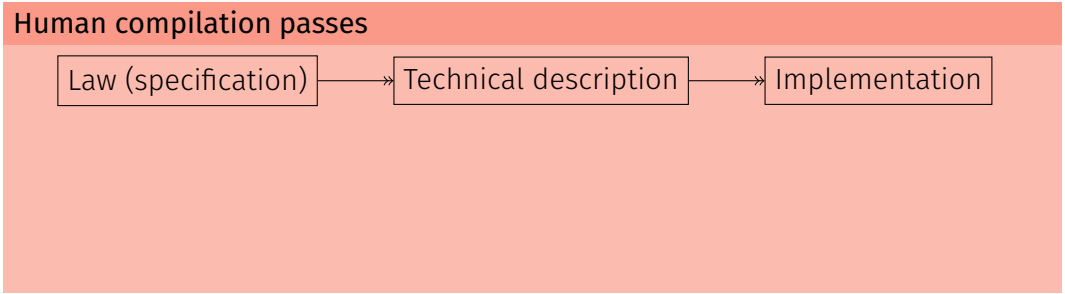
`https://gitlab.adullact.net/dgfip/impots-nationaux-revenu-patrimoine-particuliers/`

Catala, a DSL for the Law


Human compilation passes
Law (specification)
Technical description
Implementation

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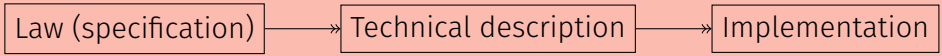

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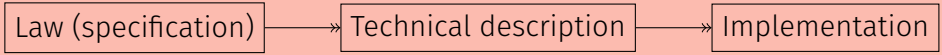

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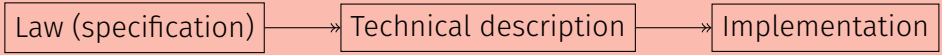

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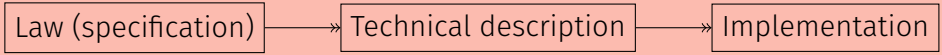

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Catala, a new DSL

Initiated through interdisciplinary work, and requiring peer programming

- Merigoux, Chataing, and Protzenko [MCP21]
- Huttner and Merigoux [HM22]
- Merigoux, Alauzen, and Slimani [MAS23]

Structure of computational law: income tax example

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For people in charge of 3 or more children, the percentage mentioned at article 1 is 15%.

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Default logic

Running Example on Income Tax

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The fixed percentage mentioned at article 1 is 20%.

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 # Article 3
If the income is less than $10,000, the percentage mentioned at article 1 is 10%.

```catala
scope IncomeTaxComputation:
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 # Article 4
For people in charge of 3 or more children, the percentage mentioned at article 1 is 15%.

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Compilation into Default Terms

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$$e ::= \langle\, e_1, \ldots, e_n \;\mid\; b_{default} \;:-\; e_{default} \,\rangle$$

Compilation into Default Terms

$$\text{Source code} \xrightarrow{\text{compiler}} \textbf{default terms}$$

$$e ::= \langle \underbrace{e_1, \ldots, e_n}_{\text{exceptions}} \mid b_{default} \ :- \ e_{default} \rangle$$

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$$\begin{array}{rcl} v & ::= & \texttt{true} \mid \texttt{false} \mid n \mid \ldots \\ & \mid & \varnothing \\ & \mid & \circledast \end{array}$$

Default Terms: Semantics

```
⟨
  ⟨ | income ≤ $10,000 :- 10%⟩,
  ⟨ | nb_children ≥ 3 :- 15%⟩
  | true :- 20%
⟩
```

```
exception definition tax_rate
 under condition house.income <= $10,000
 consequence equals 10%
exception definition tax_rate
 under condition house.nb_children >= 3
 consequence equals 15%
definition tax_rate equals 20%
```

Default Terms: Semantics

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\begin{aligned}
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1. Evaluate all exceptions
2. If **exactly 1 exception** is raised, then **return its value**
3. Else if **at least 2 exceptions** are raised, then **return** $\circledast$
4. Else if **0 exceptions** are raised, evaluate $b_{default}$ and
   - If $b_{default} = \texttt{true}$, then **evaluate** $e_{default}$
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Default Terms: Semantics

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Catala Examples

Multiple examples and prototypes developed by peer-programming processes

Name	Country	Combined LOC
Minimum wage	FR	490
Tax Code (S. 121, 132)	US	637
Family benefits	FR	2,044
Housing benefits	FR	25,290

`https://github.com/CatalaLang/catala-examples/`

Article D823-20 of the French building regulations

The moving allowance is awarded to individuals or households with at least three children born or to be born and who move into a new home entitled to one of the personal housing allowances during a period between the first day of the calendar month following the third month of pregnancy for a child of rank three or more and the last day of the month preceding that in which the child reaches his or her second birthday.
This allowance is payable if the right to assistance is acquired within six months of the date of moving in.

```catala
scope MovingAllowanceEligibility:
 definition condition_moving_period under condition
  (match form.birthdate_third_child_or_more with pattern
  -- MoreThan3Children of date_of_birth_or_pregnancy:
  (match date_of_birth_or_pregnancy with pattern
    --
    --
    -- DateOfBirth of birthday: current_date < (first_day_of_month of (birthday + 2 y))
   )
  )
consequence fulfilled
```

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  (match date_of_birth_or_pregnancy with pattern
    -- BeforeFirstDayOfThirdMonthPregnancy: false
    -- AfterFirstDayOfThirdMonthPregnancy: true
    -- DateOfBirth of birthday: current_date < (first_day_of_month of (birthday + 2 y))
   )
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consequence fulfilled
```

Outlook: Public Administration Adoption (Merigoux, Gesbert, Primet, Botbol)

Public administration trials

- DGFIP Income Tax Experiment, Jul. 2023-Oct. 2024, 8.9kLOC
- CNAF Family Benefits, Oct. 2024-Oct. 2025, 8.4kLOC

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Inria Press Release About Trials

Towards adoption at CNAF

CNAF CIO interview

- Emphasis on sovereignty (compared to proprietary solutions), open-source
- Housing benefits, solidarity income, in-work benefit will be migrated

Automated Verification of Catala Programs

Catala is safe

- Designed by PL specialists
- Statically typed
- Decimals for money
- Not Turing complete

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- **Ambiguities** in the code

$\rightsquigarrow$ RTEs unwanted for large batch computations

Ambiguities in Catala code

```
# Article 3
If the income is less than $10,000, the
percentage mentioned at article 1 is 10%.
```catala
scope IncomeTaxComputation:
 exception definition tax_rate
  under condition house.income <= $10,000
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```
# Article 4
For people in charge of 3 or more children,
the percentage mentioned at article 1 is 15%.
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scope IncomeTaxComputation:
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Ambiguities in Catala code

- interpretation conflicts, *e.g.* `income = $9,000` and `children = 4`

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Ambiguities in Catala code

- interpretation conflicts, *e.g.* `income = $9,000` and `children = 4`
- unhandled cases
- resolved by lawyers/administration if implementation is correct

Fixing the interpretation conflict

Suppose the lawyer says the `income` condition has priority.

```
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Fixing the interpretation conflict

Suppose the lawyer says the `income` condition has priority.
$\rightarrow$ it becomes an exception to the exception.

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  ⟨ | income ≤ $10,000 :- 10%⟩,
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Automated Verification of Catala Programs

Date Arithmetic [MFM24]

```
$ date -d "2024-01-31 + 1 month" +%F
```

```
$ date -d "2024-01-31 + 1 month" +%F
2024-03-02
```

```
$ date -d "2024-01-31 + 1 month" +%F
2024-03-02
$ date -d "2024-02-01 + 1 month" +%F
```

```
$ date -d "2024-01-31 + 1 month" +%F
2024-03-02
$ date -d "2024-02-01 + 1 month" +%F
2024-03-01
```

```
$ date -d "2024-01-31 + 1 month" +%F
2024-03-02
$ date -d "2024-02-01 + 1 month" +%F
2024-03-01
```

Non-monotonic behavior?!

A wide variety of choices

Different legal bodies and choices

- 1 month = 30 days (Council of European Communities)

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$\Longrightarrow$ Formal, flexible semantics required!

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- 1 month = 30 days (Council of European Communities)
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$\Longrightarrow$ Formal, flexible semantics required! Focus on Gregorian calendar.

Semantics – Values

values	v	$::=$	$(y, m, d) \mid \bot$
date unit	δ	$::=$	$y \mid m \mid d$
expressions	e	$::=$	$v \mid e \ +_\delta \ n$

Semantics – Values

$$
\begin{array}{lrcl}
\text{values} & v & ::= & (y, m, d) \mid \bot \\
\text{date unit} & \delta & ::= & y \mid m \mid d \\
\text{expressions} & e & ::= & v \mid e \ +_\delta \ n
\end{array}
$$

$$
\text{nb\_days}(y, m) = \begin{cases}
29 \text{ if } m = \texttt{Feb} \wedge \text{is\_leap}(y) \\
28 \text{ if } m = \texttt{Feb} \wedge \neg\text{is\_leap}(y) \\
30 \text{ if } m \in \{ \texttt{Apr}, \texttt{Jun}, \texttt{Sep}, \texttt{Nov} \} \\
31 \text{ otherwise}
\end{cases}
$$

Semantics – Values

$$
\begin{array}{lrcl}
\text{values} & v & ::= & (y, m, d) \mid \bot \\
\text{date unit} & \delta & ::= & y \mid m \mid d \\
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31 \text{ otherwise}
\end{cases}
$$

$$
\text{valid}(v) \Leftrightarrow v \neq \bot \wedge v = (y, m, d) \wedge 1 \leq d \leq \text{nb\_days}(y, m)
$$

Semantics – invalid day number

Day additions with invalid day number propagate errors

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$$\textsc{Add-Days-Err1}\ \frac{day < 1}{(y, m, day) +_d n \to \bot}$$

Semantics – invalid day number

Day additions with invalid day number propagate errors

$$\textsc{Add-Days-Err1}\quad \frac{day < 1}{(y, m, day) +_d n \to \bot} \qquad \textsc{Add-Days-Err2}\quad \frac{day > \text{nb\_days}(y, m)}{(y, m, day) +_d n \to \bot}$$

Semantics – some cases of month addition

ADD-MONTH

$$\frac{1 \leq mo + n \leq 12}{(y, mo, d) +_m n \to (y, mo + n, d)}$$

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$$\frac{1 \leq mo + n \leq 12}{(y, mo, d) +_m n \to (y, mo + n, d)}$$

ADD-MONTH-OVER

$$\frac{mo + n > 12}{(y, mo, d) +_m n \to (y + 1, mo, d) +_m (n - 12)}$$

Semantics – some cases of month addition

ADD-MONTH

$$\frac{1 \leq mo + n \leq 12}{(y, mo, d) +_m n \to (y, mo + n, d)}$$

ADD-MONTH-OVER

$$\frac{mo + n > 12}{(y, mo, d) +_m n \to (y + 1, mo, d) +_m (n - 12)}$$

Similar cases for ADD-MONTH-UNDER, year, day addition.

Semantics – Rounding

$$(2024, 01, 31) +_m 1 \to (2024, 02, 31)$$

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Rounding to valid dates required!

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Rounding to valid dates required!

$$\begin{array}{lrcl} \text{rounding mode} & r & ::= & \uparrow \mid \downarrow \mid \bot \\ \text{expressions} & e & ::= & v \mid e +_\delta n \mid \mathrm{rnd}_r\, e \end{array}$$

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$$\mathrm{rnd}_\uparrow(2024, 02, 31) = (2024, 03, 01)$$

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$$\mathsf{rnd}_\bot(2024, 02, 31) = \bot$$

$$(2024, 01, 31) +_m 1 \to (2024, 02, 31)$$

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$$\mathsf{rnd}_\uparrow(2024, 02, 31) = (2024, 03, 01)$$
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$$\mathsf{rnd}_\bot(2024, 02, 31) = \bot$$

Coreutils-like rounding not defined here

ROUND-NOOP

$$\frac{1 \leq d \leq \text{nb\_days}(y, m)}{\text{rnd}_r(y, m, d) \to (y, m, d)}$$

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ROUND-DOWN

$$\frac{d > \text{nb\_days}(y, m)}{\text{rnd}_\downarrow(y, m, d) \to (y, m, \text{nb\_days}(y, m))}$$

Semantics – Rounding

ROUND-NOOP

$$\frac{1 \leq d \leq \text{nb\_days}(y, m)}{\text{rnd}_r(y, m, d) \to (y, m, d)}$$

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$$\frac{d > \text{nb\_days}(y, m)}{\text{rnd}_\downarrow(y, m, d) \to (y, m, \text{nb\_days}(y, m))}$$

ROUND-UP

$$\frac{d > \text{nb\_days}(y, m) \qquad (y, m, d) +_m 1 \xrightarrow{*} (y', m', d')}{\text{rnd}_\uparrow(y, m, d) \to (y', m', 1)}$$

ROUND-NOOP

$$\frac{1 \leq d \leq \text{nb\_days}(y, m)}{\text{rnd}_r(y, m, d) \to (y, m, d)}$$

ROUND-DOWN

$$\frac{d > \text{nb\_days}(y, m)}{\text{rnd}_\downarrow(y, m, d) \to (y, m, \text{nb\_days}(y, m))}$$

ROUND-UP

$$\frac{d > \text{nb\_days}(y, m) \qquad (y, m, d) +_m 1 \xrightarrow{*} (y', m', d')}{\text{rnd}_\uparrow(y, m, d) \to (y', m', 1)}$$

ROUND-ERR2

$$\frac{d > \text{nb\_days}(y, m)}{\text{rnd}_\bot(y, m, d) \to \bot}$$

Date-period addition

Given a period (ys, ms, ds):

$$e +_r (ys, ms, ds) ::= \mathrm{rnd}_r((e +_y ys) +_m ms) +_d ds$$

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Given a period (ys, ms, ds):

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Avoids double rounding

Ambiguous expression

A date expression e is ambiguous iff $\mathrm{rnd}_\perp(e) \xrightarrow{*} \perp$

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Ambiguous expression

A date expression e is ambiguous iff $\mathrm{rnd}_\perp(e) \xrightarrow{*} \perp$

iff roundings e yield different values

Non-properties

Commutativity of addition

$$(2024, 03, 31) +_{\uparrow} 1\mathrm{m} +_{\uparrow} 1\mathrm{d} = (2024, 05, 01) +_{\uparrow} 1\mathrm{d} = (2024, 05, 02)$$

Non-properties

Commutativity of addition

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$$(2024, 03, 31) +_{\uparrow} 1\text{d} +_{\uparrow} 1\text{m} = (2024, 04, 01) +_{\uparrow} 1\text{m} = (2024, 05, 01)$$

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Commutativity of addition

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$$(2024, 03, 31) +_{\uparrow} 1\text{d} +_{\uparrow} 1\text{m} = (2024, 04, 01) +_{\uparrow} 1\text{m} = (2024, 05, 01)$$

“Associativity” of addition

$$(2024, 03, 31) +_{\uparrow} 1\text{m} +_{\uparrow} 1\text{m} = (2024, 05, 01) +_{\uparrow} 1\text{m} = (2024, 06, 01)$$

Non-properties

Commutativity of addition

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$$(2024, 03, 31) +_{\uparrow} 1\text{d} +_{\uparrow} 1\text{m} = (2024, 04, 01) +_{\uparrow} 1\text{m} = (2024, 05, 01)$$

“Associativity” of addition

$$(2024, 03, 31) +_{\uparrow} 1\text{m} +_{\uparrow} 1\text{m} = (2024, 05, 01) +_{\uparrow} 1\text{m} = (2024, 06, 01)$$
$$(2024, 03, 31) +_{r} 2\text{m} = (2024, 05, 31)$$

Formalized properties

All formalized with the F* proof assistant.

- More in the paper [MFM24].

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Date addition is monotonic

For any dates d_1, d_2, period p, $r \in \{\downarrow, \uparrow\}$, if $d_1 < d_2$, then $d_1 +_r p \leq d_2 +_r p$

Meaningful ambiguities

Rounding choice can change comparisons

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$\mathsf{d}(d_1) \in [1, 31] \wedge \mathsf{m}(d_1) \in [1, 12] \wedge \mathsf{y}(d_1) = 2024$: all valid dates of 2024

Transfer function computing $(\mathtt{d}, \mathtt{m}, \mathtt{y}) +^{\#} \mathtt{nb\_m}$ in abstract state `abs`

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let add_months ((d, m, y): var^3) (nb_m: int) (abs: state) =
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    (* ~> continuation (assume guard) *)



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    mk_true, mk_date d r_m r_y;
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Moving to double programs

- Analyze the program in both rounding modes

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Implementation into Mopsa 

- Open-source static analysis platform

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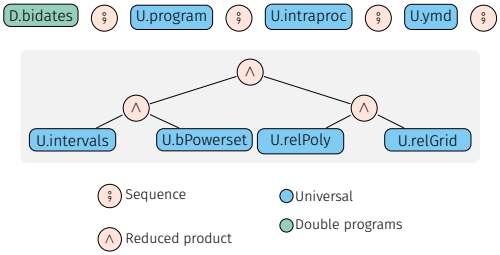

Extracted sample from French housing benefits

```
date current = rand_date();
date birthday = rand_date();
date intermediate = birthday + [2 years, 0 months, 0 days];
date limit = first_day_of(intermediate);
assert(sync(current < limit));
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```
5: assert(sync(current < limit));
                ^^^^^^^^^^^^^^^^
```

```
Desynchronization detected: (current < limit). Hints:
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↑month(intermediate) = 3, ↑day(intermediate) = 1, ↓month(intermediate) = 2,
↓day(intermediate) = 28, month(birthday) = 2, day(birthday) = 29,
year(birthday) =[4] 0, month(current) = 2, day(current) = [1,29],
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Computed, actual counter-example

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Contributions to Catala

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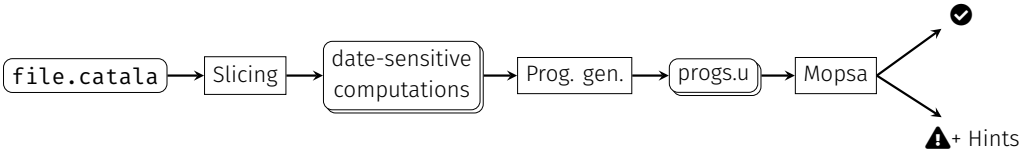
Date ambiguity detection pipeline



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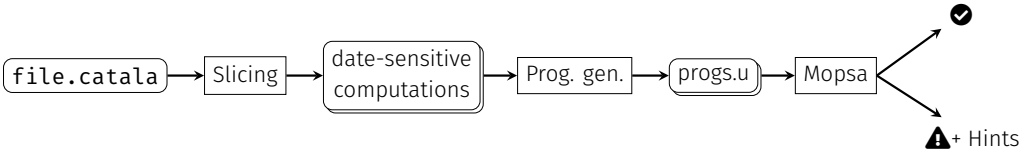

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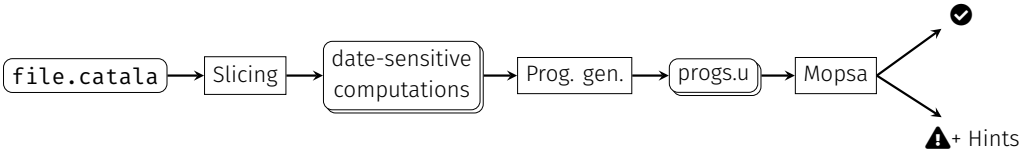

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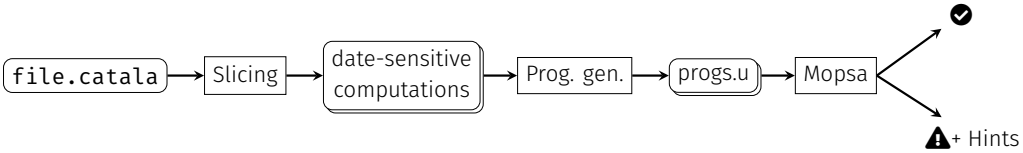

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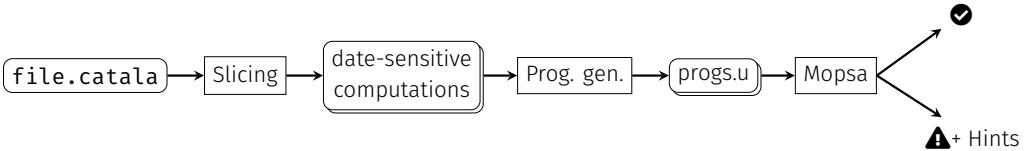

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Intra-scope extraction for now

Manual inter-scope extraction

16 additional cases:

- 10 can be proved safe assuming `current_date` $\geq$ `2023`
- Other are real issues

What about mainstream implementations?

Survey of implementations

- Java, `boost` round down
- Python `stdlib`: no month addition
- `boost` C++ rounds down
- Rounding inconsistency in spreadsheet operators

Automated Verification of Catala Programs

CuteCat Concolic Execution [GFM25]

Concolic execution: first example

```
if x > 0
    then 0
    else if y < 0
        then 1
        else error
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```
if x > 0
```

Step	x	y	Output	Constraints after evaluation	Next path to try

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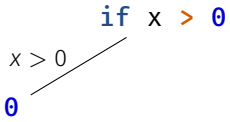

Step	x	y	Output	Constraints after evaluation	Next path to try
1	1	20	0	$x > 0$	

Concolic execution: first example

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if x > 0
    then 0
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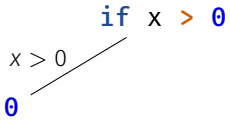
```



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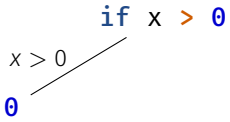

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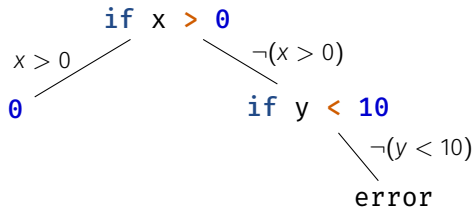

Step	x	y	Output	Constraints after evaluation	Next path to try	
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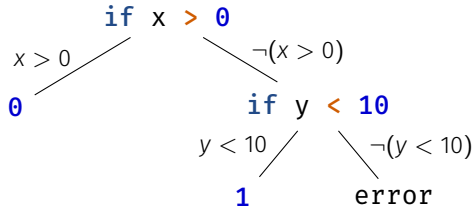

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3	0	9	1	$\neg(x > 0) \wedge y < 10$	-	

Concolic execution of default terms

$$\langle\langle \mid \texttt{income} \leq \$10,000 \texttt{ :- } 10\% \rangle, \langle \mid \texttt{nb\_children} \geq 3 \texttt{ :- } 15\% \rangle \mid \texttt{true} \texttt{ :- } 20\% \rangle$$

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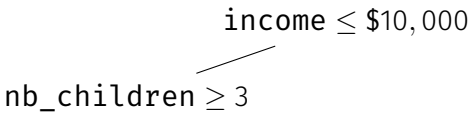

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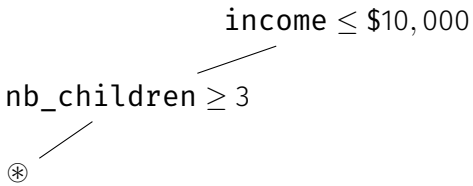

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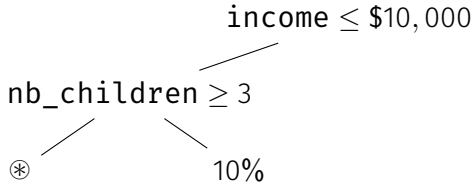

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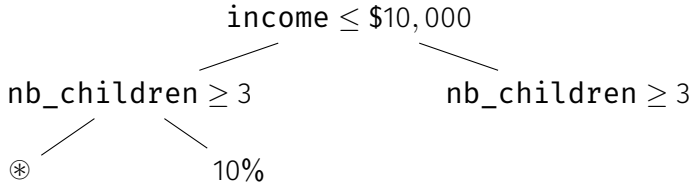
`income` = **$11,000**; `nb_children` = **4**



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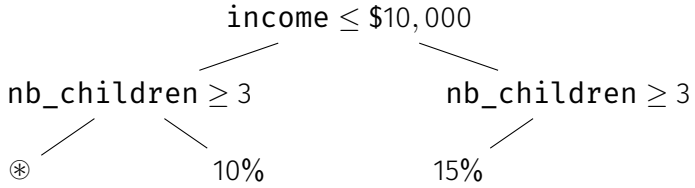

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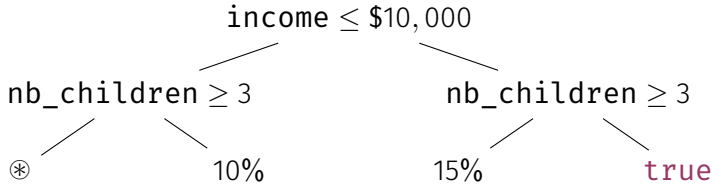

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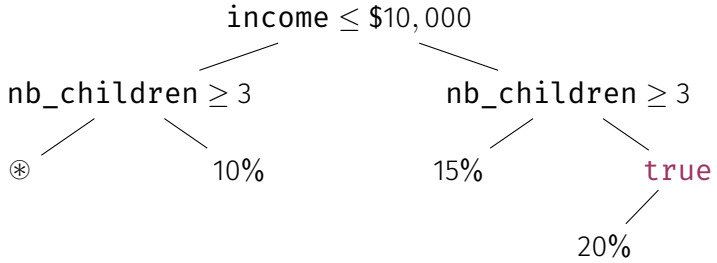

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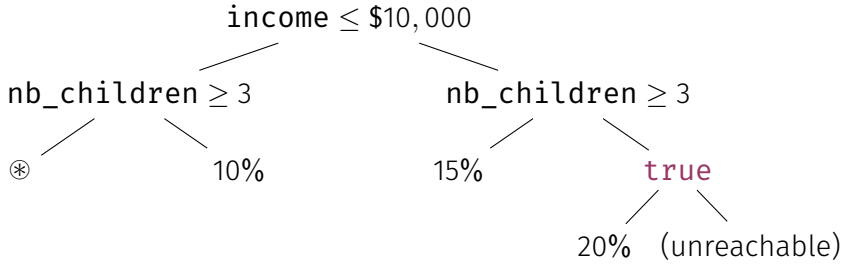

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- Possible incompleteness due to mixed integer/rational reasoning

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- Possible incompleteness due to mixed integer/rational reasoning
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Performance optimizations using reordering

Theorem (Independence of exception evaluation order)

If there is a default value v such that

$$\langle \ldots, e_i, \ldots, \mathbf{e_j}, \ldots \mid b_{default} \text{ :- } e_{default} \rangle \longrightarrow^* v,$$

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Performance optimizations using reordering – Example

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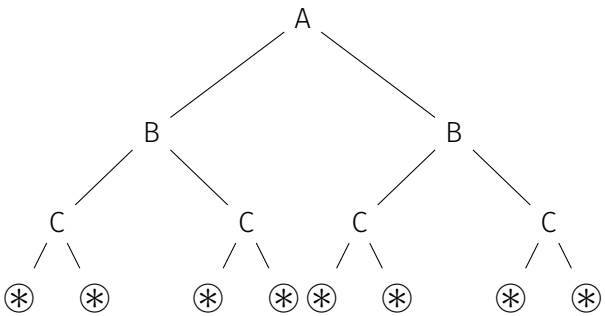

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# Article 3
If the income is less than $10,000, the percentage
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```catala
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- Find more usable input values using soft constraints !
  - *e.g.* **round to $1,000**

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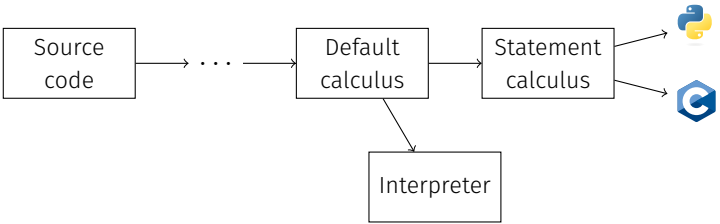
Implementation of CuteCat



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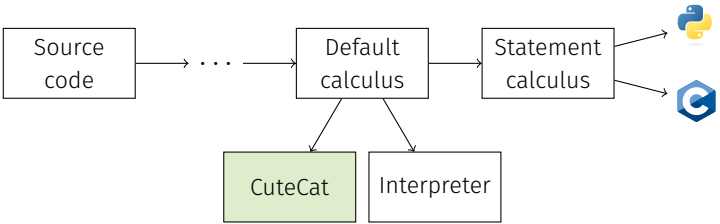

- Integrated into Catala compiler's *default calculus* IR
- 3.4k lines of OCaml code
- **Z3** SMT Solver

Case study: housing benefits

French housing benefits: **5736** lines of law, **8615** lines of code

- 186,390 test cases generated in **7h of single-threaded CPU time**
- 366s spent in solver, the rest in evaluation
- Able to find a conflict: bedrooms vs flat sharing
- 99.83% of tests satisfy soft constraints

Overhead of the concolic execution

- 4.5x overhead w.r.t. Catala interpreter on our benchmarks
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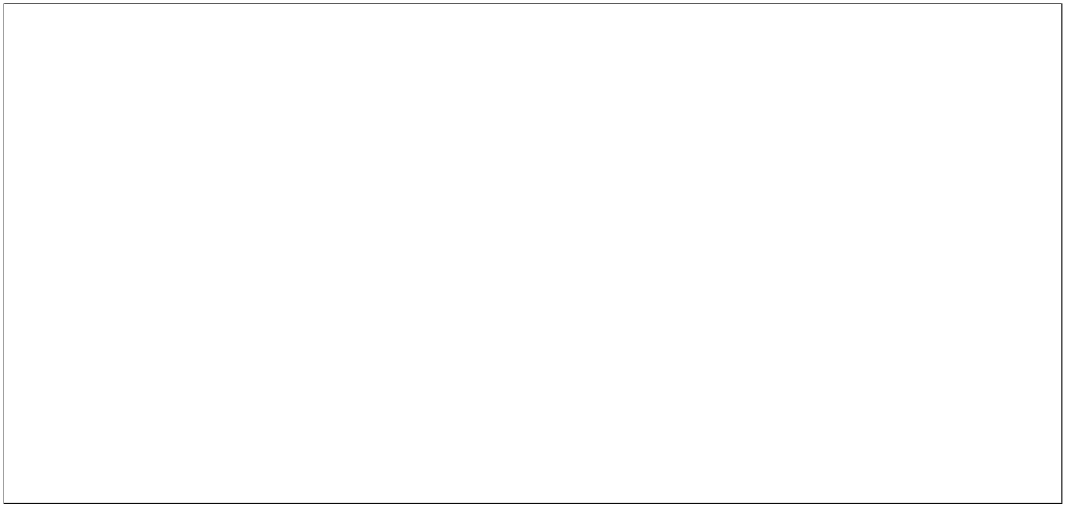

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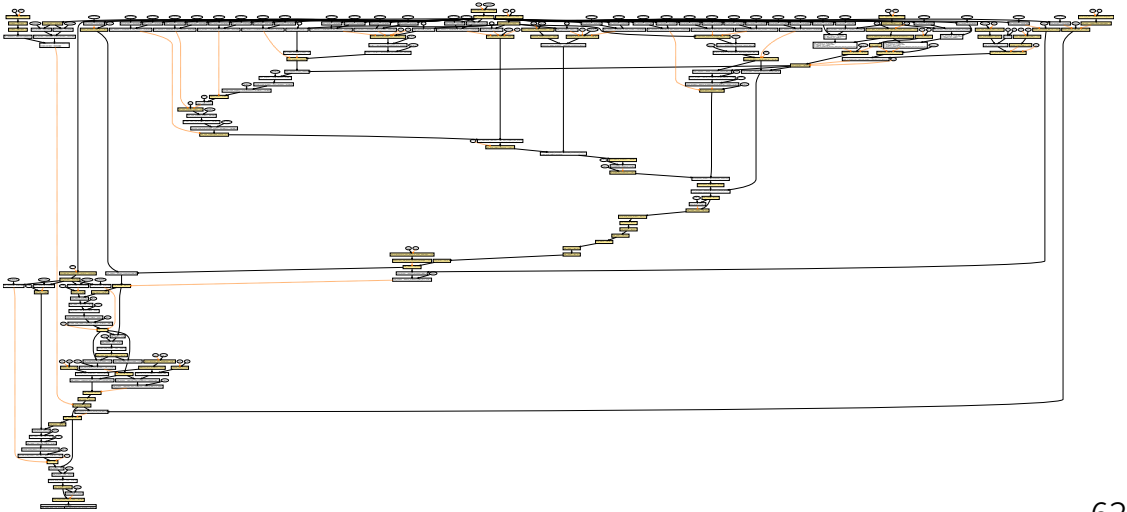

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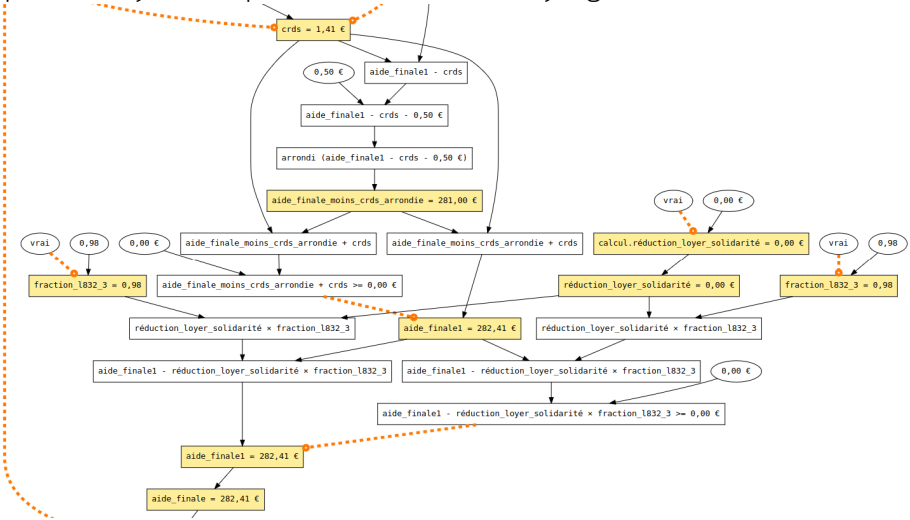

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References – I

[Ana+24] de Almeida Borges Ana et al. **“UTC Time, Formally Verified”.** In: ACM, 2024, pp. 2–13.

[Bal+19] Clément Ballabriga et al. **“Static Analysis of Binary Code with Memory Indirections Using Polyhedra”.** In: Lecture Notes in Computer Science. Springer, 2019, pp. 114–135.

[Bal19] Nathan Ballantyne. **“Epistemic trespassing”.** In: Mind 510 (2019), pp. 367–395.

[Bel21] Marianne Bellotti. **Kill it with fire: manage aging computer systems.** No Starch Press, 2021.

References – II

[Che+22] Ju Chen et al. “SYMSAN: Time and Space Efficient Concolic Execution via Dynamic Data-Flow Analysis”. In: USENIX Association, 2022, pp. 2531–2548.

[CTP26] Leyi Cui, Shrey Tiwari, and Rohan Padhye. DateSAT: A Framework for Solving Date and Period Constraints. 2026.

[DOM21] David Delmas, Abdelraouf Ouadjaout, and Antoine Miné. “Static Analysis of Endian Portability by Abstract Interpretation”. In: Lecture Notes in Computer Science. Springer, 2021, pp. 102–123.

References – III

[Dut20] Dutch Data Protection Authority. Werkwijze Belastingdienst in strijd met de wet en discriminerend. 2020. URL: https://autoriteitpersoonsgegevens.nl/actueel/werkwijze-belastingdienst-in-strijd-met-de-wet-en-discriminerend.

[EB20] Nel Escher and Nikola Banovic. "Exposing Error in Poverty Management Technology: A Method for Auditing Government Benefits Screening Tools". In: Proc. ACM Hum. Comput. Interact. CSCW (2020), 064:1–064:20.

[Esc+24] Nel Escher et al. "Code-ifying the Law: How Disciplinary Divides Afflict the Development of Legal Software". In: Proc. ACM Hum. Comput. Interact. CSCW2 (2024), pp. 1–37. DOI: 10.1145/3686937.

References – IV

[GFM25] Pierre Goutagny, Aymeric Fromherz, and Raphaël Monat. **“CUTECat: Concolic Execution for Computational Law”.** In: ed. by Viktor Vafeiadis. Lecture Notes in Computer Science. Springer, 2025, pp. 31–61. DOI: 10.1007/978-3-031-91121-7_2.

[HM22] Liane Huttner and Denis Merigoux. **“Catala: Moving Towards the Future of Legal Expert Systems”.** In: Artificial Intelligence and Law (Aug. 2022). DOI: 10.1007/s10506-022-09328-5.

[HS24] Felienne Hermans and Ari Schlesinger. **“A Case for Feminism in Programming Language Design”.** In: ed. by Jonathan Edwards and Marcel Taeumel. ACM, 2024, pp. 205–222. DOI: 10.1145/3689492.3689809.

References – V

[Law24] Sarah Lawsky. **“Computational Law and Epistemic Trespassing”.** In: Journal of Cross-disciplinary Research in Computational Law 2 (May 2024).

[MAS23] Denis Merigoux, Marie Alauzen, and Lilya Slimani. **“Rules, Computation and Politics. Scrutinizing Unnoticed Programming Choices in French Housing Benefits”.** In: Journal of Cross-disciplinary Research in Computational Law 1 (2023), p. 23.

[MCP21] Denis Merigoux, Nicolas Chataing, and Jonathan Protzenko. **“Catala: a programming language for the law”.** In: 2021.

[Mer+24] Denis Merigoux et al. De la transparence à l’explicabilité automatisée des algorithmes : comprend Tech. rep. RR-9535. INRIA Paris, Jan. 2024, p. 68.

References – VI

[MFM24] Raphaël Monat, Aymeric Fromherz, and Denis Merigoux. **“Formalizing Date Arithmetic and Statically Detecting Ambiguities for the Law”.** In: ed. by Stephanie Weirich. Lecture Notes in Computer Science. Springer, 2024, pp. 421–450. DOI: 10.1007/978-3-031-57267-8_16.

[MH26] Raphaël Monat and Liane Huttner. **“Coding computational laws: 20 recommendations for public administrations”.** In: Information & Communications Technology Law 0 (2026), pp. 1–17. DOI: 10.1080/13600834.2026.2628389.

References – VII

[MMP21] Denis Merigoux, Raphaël Monat, and Jonathan Protzenko. **“A modern compiler for the French tax code”.** In: ed. by Aaron Smith, Delphine Demange, and Rajiv Gupta. ACM, 2021, pp. 71–82. DOI: 10.1145/3446804.3446850.

[Moc18] Percy Mockler. The Phoenix Pay Problem: Working toward a Solution. Senate Canada, 2018.

[PF20] Sebastian Poeplau and Aurélien Francillon. **“Symbolic Execution with SymCC: Don’t Interpret, Compile!”** In: SEC’20. USA: USENIX Association, 2020, pp. 181–198.

[Red+22] J Redden et al. **“Automating public services: learning from cancelled systems”.** In: Cardiff, UK: Data Justice Lab, Cardiff University (2022).

References – VIII

[Tiw+25] Shrey Tiwari et al. **“It’s About Time: An Empirical Study of Date and Time Bugs in Open-Source Python Software”.** In: 2025, pp. 39–51. DOI: 10.1109/MSR66628.2025.00020.

[Yun+18] Insu Yun et al. **“QSYM : A Practical Concolic Execution Engine Tailored for Hybrid Fuzzing”.** In: USENIX Association, 2018, pp. 745–761.