Formal methods meet legal implementations

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GT LVP, ENS Paris-Saclay
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Legal implementations?

Software implementations running laws precisely describing computations

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- Income tax

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- Payroll systems

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- Social benefits
- Payroll systems
- ~~AI judges~~

Legal implementations are critical software

Critical software with tremendous impact on citizens

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Need more?

See “Automating public services: learning from cancelled systems” [Red+22].

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  - CS or CS+Law teams
  - "need for greater skepticism about the [...] reliability of legal software"
  - "participants (esp. CS) expressed unwarranted confidence in [tool utility]"

Legal implementations are updated regularly and incrementally

$\simeq$ yearly legislative changes needing implementations

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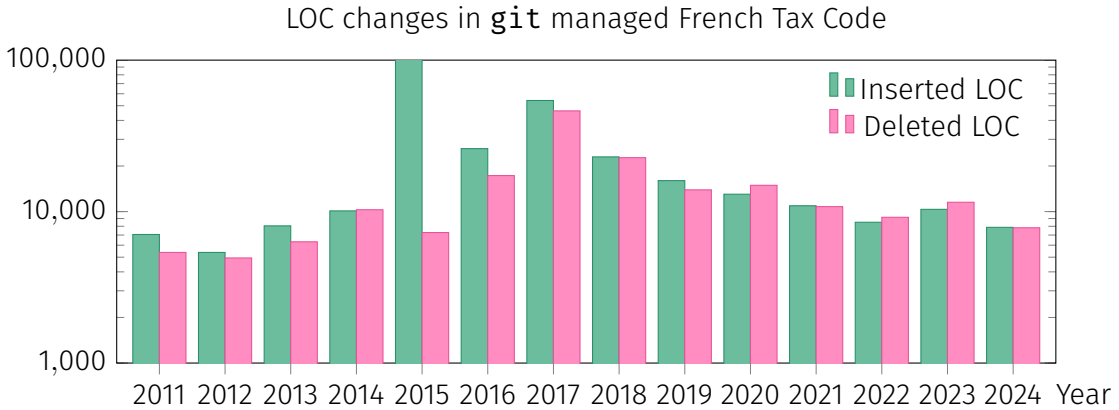

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French Income Tax

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French Benefits

- 1996 Centralization of multiple benefits
- Modernization efforts, types of benefits has tripled

Towards high-assurance legal implementations

Trusting the process?

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- Reproducibility of the computation?

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Trusting the process?

- Reproducibility of the computation?
- Compliance with the law, acting as specification?
- Usability by public administrations, citizens, NGOs, researchers?
- Accurate simulations?

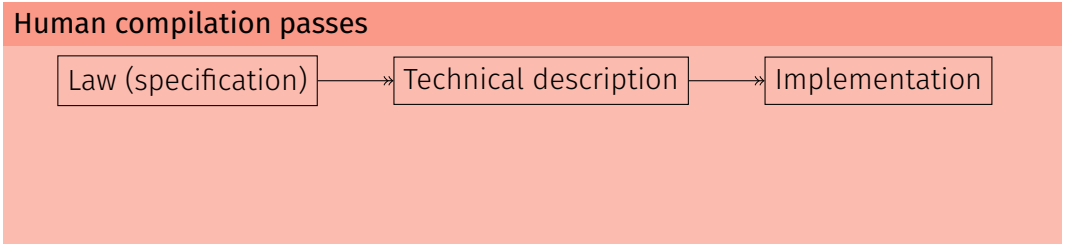
Outline

1. Introduction
2. Catala, a DSL for the Law
3. Automated Verification of Catala Programs
   - Date Arithmetic [MFM24]
   - CuteCat Concolic Execution [GFM25]
4. Conclusion

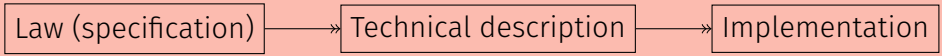
Joint work

- Marie Alauzen
- Justine Banuls
- Vincent Botbol
- Nicolas Chataing
- Caroline Flori
- Aymeric Fromherz
- Louis Gesbert
- Pierre Goutagny (slides ack)
- Estelle Hary
- Liane Huttner
- Sarah Lawsky
- Denis Merigoux (now transfer lead)
- Romain Primet
- Jonathan Protzenko
- Émile Rolley
- Lilya Slimani

Catala, a DSL for the Law


Human compilation passes
Law (specification)
Technical description
Implementation

Human compilation passes



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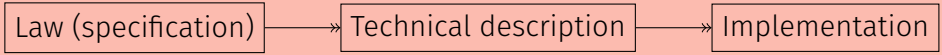

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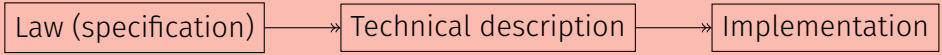

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Catala, a new DSL

Initiated through interdisciplinary work, and requiring peer programming

- Merigoux, Chataing, and Protzenko [MCP21]
- Huttner and Merigoux [HM22]
- Merigoux, Alauzen, and Slimani [MAS23]

Structure of computational law: income tax example

Article 1

The income tax is a fixed percentage of the income.

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Article 3

If the income is less than $10,000, the percentage mentioned at article 1 is 10%.

Article 4

For people in charge of 3 or more children, the percentage mentioned at article 1 is 15%.

Structure of computational law: income tax example

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Article 3 exception

If the income is less than $10,000, the percentage mentioned at article 1 is 10%.

Article 4 exception

For people in charge of 3 or more children, the percentage mentioned at article 1 is 15%.

Default logic

Running Example on Income Tax

```
# Article 1
The income tax is a fixed percentage of
the income.
```catala
scope IncomeTaxComputation:
 definition income_tax equals
   house.income * tax_rate
```

Article 2
The fixed percentage mentioned at
article 1 is 20%.
```catala
scope IncomeTaxComputation:
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```

```
# Article 3
If the income is less than $10,000, the
percentage mentioned at article 1 is 10%.
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```

Article 4
For people in charge of 3 or more
children, the percentage mentioned at
article 1 is 15%.
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scope IncomeTaxComputation:
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https://book.catala-lang.org/playground/learn.html?chapter=2-1-basic-blocks

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Compilation into Default Terms

Source code $\xrightarrow{\text{compiler}}$ **default terms**

$$e ::= \langle\, e_1, \ldots, e_n \;\mid\; b_{default} \;:-\; e_{default} \,\rangle$$

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$$\begin{array}{rcl} v & ::= & \texttt{true} \mid \texttt{false} \mid n \mid \ldots \\ & \mid & \varnothing \\ & \mid & \circledast \end{array}$$

Default Terms: Semantics

```
⟨
  ⟨ | income ≤ $10,000 :- 10%⟩,
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  | true :- 20%
⟩
```

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exception definition tax_rate
 under condition house.income <= $10,000
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2. If **exactly 1 exception** is raised, then **return its value**
3. Else if **at least 2 exceptions** are raised, then **return** $\circledast$
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Public administration trials

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- CNAF Family Benefits, Oct. 2024-Oct. 2025, 8.4kLOC

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Towards adoption at CNAF

CNAF CIO interview

- Emphasis on sovereignty (compared to proprietary solutions), open-source
- Housing benefits, solidarity income, in-work benefit will be migrated

Automated Verification of Catala Programs

Catala is safe

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- Statically typed
- Decimals for money
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But runtime errors can still abound

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Motivation

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⇝ RTEs unwanted for large batch computations

Ambiguities in Catala code

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```
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```
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Ambiguities in Catala code

- interpretation conflicts, *e.g.* `income = $9,000` and `children = 4`
- unhandled cases
- resolved by lawyers/administration if implementation is correct

Fixing the interpretation conflict

Suppose the lawyer says the `income` condition has priority.

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```

Fixing the interpretation conflict

Suppose the lawyer says the `income` condition has priority.
$\rightarrow$ it becomes an exception to the exception.

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⟩
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Automated Verification of Catala Programs

Date Arithmetic [MFM24]

```
$ date -d "2024-01-31 + 1 month" +%F
```

Motivation

```
$ date -d "2024-01-31 + 1 month" +%F
2024-03-02
```

Motivation

```
$ date -d "2024-01-31 + 1 month" +%F
2024-03-02
$ date -d "2024-02-01 + 1 month" +%F
```

```
$ date -d "2024-01-31 + 1 month" +%F
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$ date -d "2024-01-31 + 1 month" +%F
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$ date -d "2024-02-01 + 1 month" +%F
2024-03-01
```

Non-monotonic behavior?!

A wide variety of choices

Different legal bodies and choices

- 1 month = 30 days (Council of European Communities)

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$\Longrightarrow$ Formal, flexible semantics required! Focus on Gregorian calendar.

Semantics – Rounding

$$(2024, 01, 31) +_m 1 \to (2024, 02, 31)$$

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$$\mathrm{rnd}_\uparrow(2024, 02, 31) = (2024, 03, 01)$$
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Coreutils-like rounding not defined here

Non-properties

Commutativity of addition

$$(2024, 03, 31) +_{\uparrow} 1\mathrm{m} +_{\uparrow} 1\mathrm{d} = (2024, 05, 01) +_{\uparrow} 1\mathrm{d} = (2024, 05, 02)$$

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Rounding choice can change comparisons

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- Will reduce the need for costly legal interpretations

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$\mathsf{d}(d_1) \in [1, 31] \wedge \mathsf{m}(d_1) \in [1, 12] \wedge \mathsf{y}(d_1) = 2024$: all valid dates of 2024

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Moving to double programs

- Analyze the program in both rounding modes

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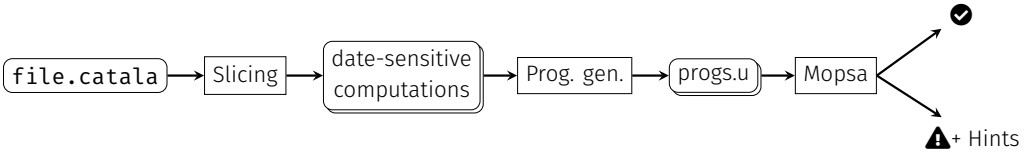
$$\uparrow\mathsf{d}(d2) = 1, \uparrow\mathsf{m}(d2) \in \{\, \mathtt{May}, \mathtt{Jul}, \mathtt{Oct}, \mathtt{Dec} \,\}, \uparrow\mathsf{m}(d2) = \mathsf{m}(d1) + 2$$
$$\downarrow\mathsf{y}(d2) = \uparrow\mathsf{y}(d2) = \mathsf{y}(d1)$$

Date ambiguity detection pipeline



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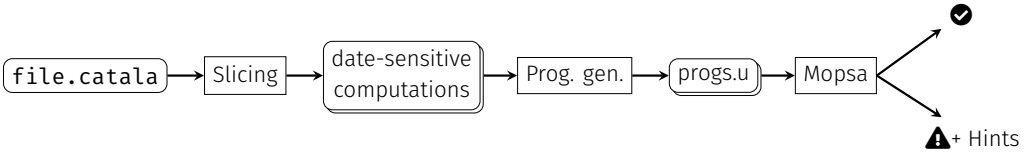

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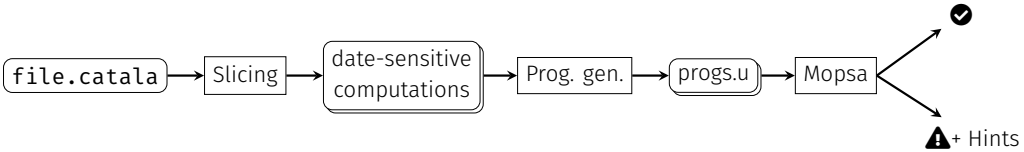

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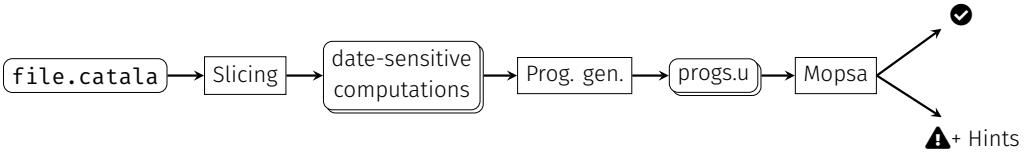

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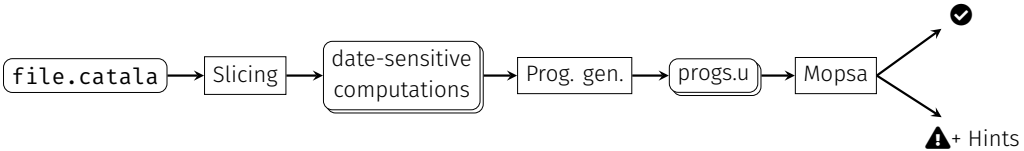

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Manual inter-scope extraction

16 additional cases:

- 10 can be proved safe assuming `current_date` $\geq$ `2023`
- Other are real issues

Automated Verification of Catala Programs

CuteCat Concolic Execution [GFM25]

Concolic execution: first example

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if x > 0
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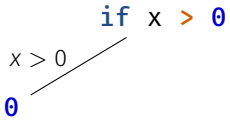

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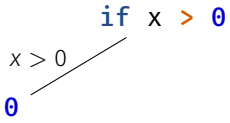

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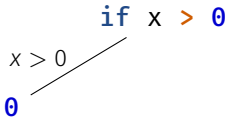

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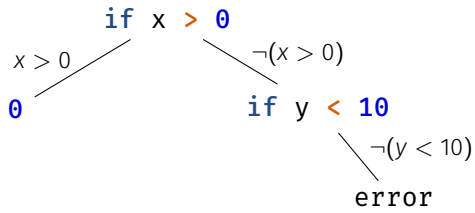

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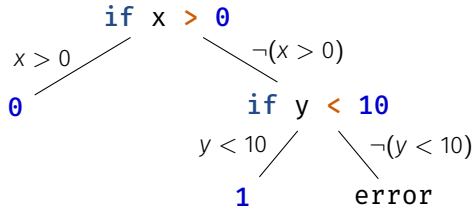

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3	0	9	1	$\neg(x > 0) \wedge y < 10$	-	

Concolic execution of default terms

$$\langle\langle \mid \texttt{income} \leq \$10,000 \texttt{ :- } 10\% \rangle, \langle \mid \texttt{nb_children} \geq 3 \texttt{ :- } 15\% \rangle \mid \texttt{true} \texttt{ :- } 20\% \rangle$$

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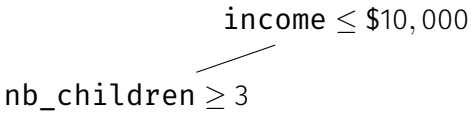

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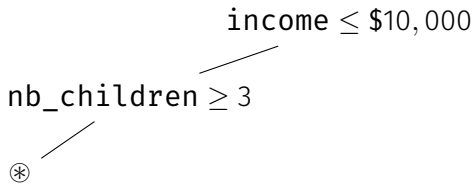

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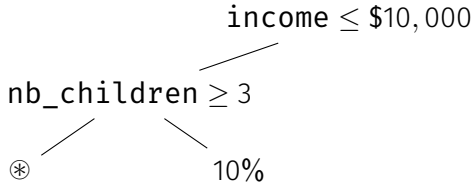

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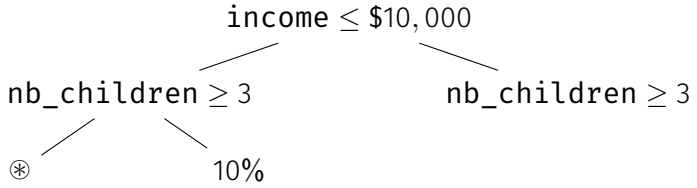
`income` = **$11,000**; `nb_children` = **4**



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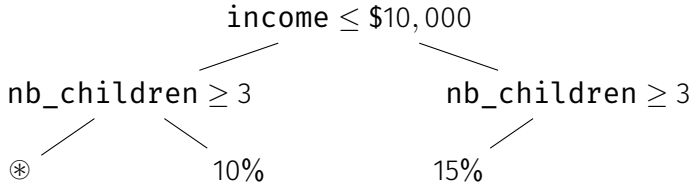

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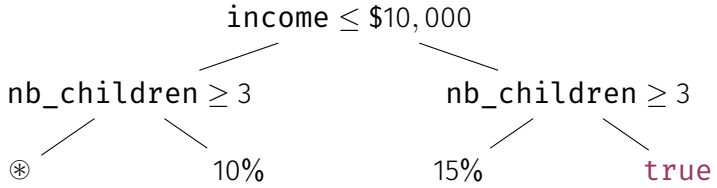

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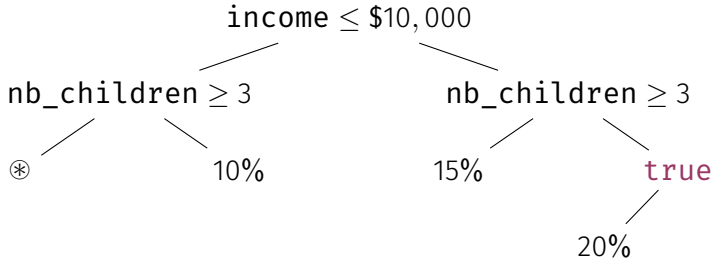
$\langle\langle \mid \texttt{income} \leq \$10,000 \texttt{ :- } 10\% \rangle, \langle \mid \texttt{nb_children} \geq 3 \texttt{ :- } 15\% \rangle \mid \texttt{true} \texttt{ :- } 20\% \rangle$

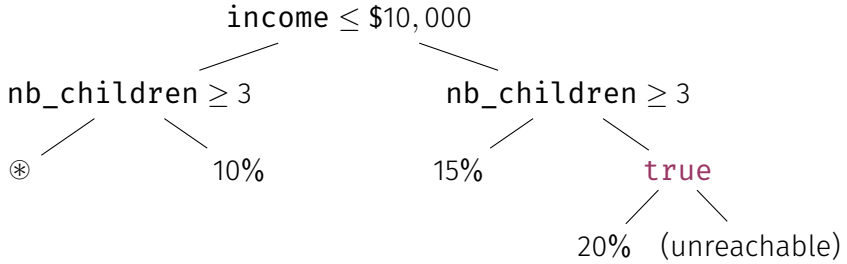
$\texttt{income} = \$11{,}000; \quad \texttt{nb_children} = 2$



Concolic execution of default terms

⟨⟨ | income ≤ $10,000 :- 10%⟩, ⟨ | nb_children ≥ 3 :- 15%⟩ | true :- 20%⟩

income = ???; nb_children = ???



Performance optimizations using reordering

Theorem (Independence of exception evaluation order)

If there is a default value v such that

$$\langle \ldots, e_i, \ldots, \mathbf{e_j}, \ldots \mid b_{default} \ :- \ e_{default} \rangle \longrightarrow^* v,$$

then

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Example:

$$\langle A, B, C, \circledast \mid b_{default} \ :- \ e_{default} \rangle \sim \langle \circledast, A, B, C \mid b_{default} \ :- \ e_{default} \rangle$$

Performance optimizations using reordering – Example

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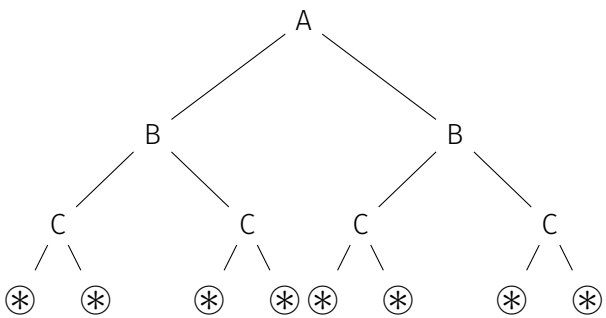



Usability improvement

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# Article 3
If the income is less than $10,000, the percentage
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scope IncomeTaxComputation:
 exception definition tax_rate
  under condition house.income <= $10,000
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- Difficult to compute by hand
- Find more usable input values using soft constraints !
  - *e.g.* **round to $1,000**

Performance optimizations of the SMT solver

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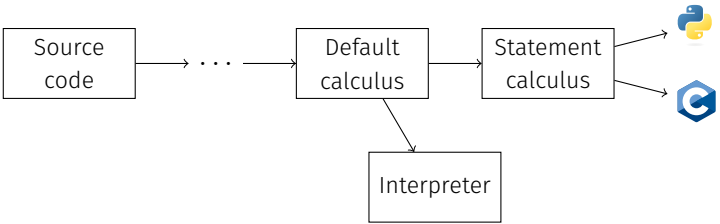
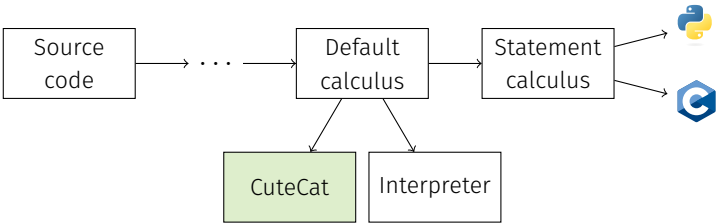
- Redundant constraint elimination

Implementation of CuteCat



Implementation of CuteCat



- Integrated into Catala compiler's *default calculus* IR
- 3.4k lines of OCaml code
- **Z3** SMT Solver

Case study: housing benefits

French housing benefits: **5736** lines of law, **8615** lines of code

- 186,390 test cases generated in **7h of single-threaded CPU time**
- 366s spent in solver, the rest in evaluation
- Able to find a conflict: bedrooms vs flat sharing
- 99.83% of tests satisfy soft constraints

Overhead of the concolic execution

- 4.5x overhead w.r.t. Catala interpreter on our benchmarks
- SymCC [PF20] or SYMSAN [Che+22] reach the same order of magnitude
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- More optimizations in future work

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- Catala is an ideal testbed for automated verification research!
- Ongoing work around “Auditing Government Benefits Screening Tools” [EB20]
- Administrations and NGOs as a “third way” between academia and industry

References – I

[Bal19] Nathan Ballantyne. **“Epistemic trespassing”.** In: Mind 510 (2019), pp. 367–395.

[Che+22] Ju Chen et al. **“SYMSAN: Time and Space Efficient Concolic Execution via Dynamic Data-Flow Analysis”.** In: USENIX Association, 2022, pp. 2531–2548.

[DOM21] David Delmas, Abdelraouf Ouadjaout, and Antoine Miné. **“Static Analysis of Endian Portability by Abstract Interpretation”.** In: Lecture Notes in Computer Science. Springer, 2021, pp. 102–123.

References – II

[Dut20] Dutch Data Protection Authority. Werkwijze Belastingdienst in strijd met de wet en discriminerend. 2020. URL: https://autoriteitpersoonsgegevens.nl/actueel/werkwijze-belastingdienst-in-strijd-met-de-wet-en-discriminerend.

[EB20] Nel Escher and Nikola Banovic. "Exposing Error in Poverty Management Technology: A Method for Auditing Government Benefits Screening Tools". In: Proc. ACM Hum. Comput. Interact. CSCW (2020), 064:1–064:20.

[Esc+24] Nel Escher et al. "Code-ifying the Law: How Disciplinary Divides Afflict the Development of Legal Software". In: Proc. ACM Hum. Comput. Interact. CSCW2 (2024), pp. 1–37. DOI: 10.1145/3686937.

References – III

[GFM25] Pierre Goutagny, Aymeric Fromherz, and Raphaël Monat. **“CUTECat: Concolic Execution for Computational Law”.** In: ed. by Viktor Vafeiadis. Lecture Notes in Computer Science. Springer, 2025, pp. 31–61. DOI: 10.1007/978-3-031-91121-7_2.

[HM22] Liane Huttner and Denis Merigoux. **“Catala: Moving Towards the Future of Legal Expert Systems”.** In: Artificial Intelligence and Law (Aug. 2022). DOI: 10.1007/s10506-022-09328-5.

[Law24] Sarah Lawsky. **“Computational Law and Epistemic Trespassing”.** In: Journal of Cross-disciplinary Research in Computational Law 2 (May 2024).

References – IV

[MAS23] Denis Merigoux, Marie Alauzen, and Lilya Slimani. **“Rules, Computation and Politics. Scrutinizing Unnoticed Programming Choices in French Housing Benefits”.** In: Journal of Cross-disciplinary Research in Computational Law 1 (2023), p. 23.

[MCP21] Denis Merigoux, Nicolas Chataing, and Jonathan Protzenko. **“Catala: a programming language for the law”.** In: 2021.

[MFM24] Raphaël Monat, Aymeric Fromherz, and Denis Merigoux. **“Formalizing Date Arithmetic and Statically Detecting Ambiguities for the Law”.** In: ed. by Stephanie Weirich. Lecture Notes in Computer Science. Springer, 2024, pp. 421–450. DOI: 10.1007/978-3-031-57267-8_16.

References – V

[Moc18] Percy Mockler. The Phoenix Pay Problem: Working toward a Solution. Senate Canada, 2018.

[PF20] Sebastian Poeplau and Aurélien Francillon. “Symbolic Execution with SymCC: Don’t Interpret, Compile!” In: SEC’20. USA: USENIX Association, 2020, pp. 181–198.

[Red+22] J Redden et al. “Automating public services: learning from cancelled systems”. In: Cardiff, UK: Data Justice Lab, Cardiff University (2022).

[Yun+18] Insu Yun et al. “QSYM : A Practical Concolic Execution Engine Tailored for Hybrid Fuzzing”. In: USENIX Association, 2018, pp. 745–761.